# MULTI-CORONAS ZERNIKE MOMENTS ON CURVELET-LIKE TRANSFORM AND APPLICATION TO PATTERN RECOGNITION

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#### ABSTRACT

The combination of Zernike moments and curvelet-like transform can bring the most significant feature coefficients in pattern recognition. Instead of using Zernike moments in the image, we apply Zernike moments on every corona of curvelet-like transform. This combination brings special properties when we can represent the shape of each the corona through Zernike moments. More especially, we use orientation of wedge of curvelet-like transform at specific scale for Zernike moments instead of using uniformly partition angle as in normal Zernike moments. The experiment on classification of sub-cellular location protein images with these coefficients has shown the advance points in comparing to normal Zernike moments in whole image.

Keywords: curvelet transform, pattern recognition, Zernike moments.

# TÓM TẮT

# Moment đa vành tròn trên miền biến đổi tựa curvelet và ứng dụng trong nhận dạng ảnh

Trong bài viết này, chúng tôi đề xuất một giải pháp trong bài toán nhận dạng mẫu thuộc lĩnh vực thị giác máy tính. Đề xuất xây dựng Zernike moments trên biến đổi trường Curvelet. Thay vì sử dụng Zernike moment trên dữ liệu ảnh trực tiếp, chúng tôi tận dụng đặc điểm tạo vành tròn của biến đổi Curvelet, sau đó áp Zernike trên từng vành để xây dựng tập đặc trưng cho tập mẫu. Như vậy, hướng của Zernike moment cũng chính là hướng của vành tròn của biến đổi curvelet. Thí nghiệm được thực hiện trên phân tích tập ảnh vi ảnh huỳnh quang cho thấy độ chính xác tương đối cao so với việc chỉ sử dụng Zernike moment gốc.

Từ khóa: biến đổi curvelet, nhận dạng pattern, Zernike moments.

#### 1. Introduction

The invention of Scanning Tunneling Microscopy and its advanced various forms have opened a new research on surface characterization at a nanometer scale. We know that nanoscale or micro-scale images usually do not contain explicit features as the other normal images such as face, landscape, etc.



Fig. 1. Some nanoscale images in semiconductor with unclear features

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The recent research on bioinformatics gains many achievements through Subcellular's Location Fluorescence (SLF) microscopy [7], [8]. The information of location proteomics can provide accuracy sub-cellular distribution of protein in given cell. Knowledge of a protein's sub-cellular distribution can contribute to a complete understanding of its function. The dynamic properties of protein sub-cellular distribution in different environmental conditions can also provide significant information about protein function. Fluorescence microscopy permits rapid collection of images with excellent resolution. It is convenient condition to analyze protein. The problem is that, we need the good feature vectors and automated classification to classify the new microscopy images based on trained samples.

Wavelet transform was an important achievement in 1980's and after that it has been applied in many fields from 1D digital signal processing to 2D image analysis. However, there are some disadvantages in 2D wavelet transform, especially to work with the curves in image. It is because the wavelet coefficients cannot represent the relationship in which they are in a line or curve in an image. Therefore, it is necessary to find a good transform that can represent a linear or curve features in 2D images. By this improvement, we can challenge to problems of nanoscale images captured in semiconductor and biology fields. With good features through coefficients of the transformations, we can move ahead to the problem of image recognition in semiconductor or biology fields. The ability to analysis the images at many scales is really essential requires. It is because, some features occurs at specific scale and disappears at other scales and vice versa. These transform with their significant coefficients must 'see' the line or curve existing in image. Together with these transformations, we also suggest the recognition or classification models to explore at most the property of multi-scales, and 'see line or curve of the coefficients'

The most significant problem in classification or recognition is set of features that represent the main properties of image or object. The 2D or 3D features are suitable for cell-level recognition. The Zernike moment, Haralick textures, wavelet, Gabor are often used for SLF recognition. Together with these features, some other features such as Euler number, morphology, fractal dimension were also developed by biology experts [1].

In this research we represent advanced points for feature set through combination of curvelet-like coefficients and Zernike moments together with multi-level SVM classification algorithm. The main idea is that, we apply Zernike moments in every corona from coarsest to finest scale of curvelet transform to represent the shape of each corona instead of whole image. After that, the values of Zernike moments in every corona will be the vector for SVM classification engine in recognition phase.

Part 2 briefly introduces localized direction multi-scale transform. Part 3 represents general Zernike moments. Part 4 represents the combination between Zernike moments and curvelet-like transform. Part 5 demonstrates the experiment results in microscopy images of HELA cells.

#### 2. The summary of curvelet transform

The curvelet transform was introduced by Candès and David Donoho [4] and upgraded by Duncan (2000) and Starck (2002) to represent the curved objects in an image. Basically, curvelet transform is based on local anisotropic multi-scale transform. Similar to Fourier, Radon, or Wavelet transform, the authors have developed the complete background in both continuous and discrete domain, and they have developed the fast algorithm for curvelet transform. The curvelet-like transforms can represent the curved features with localization and anisotropic properties. These methods use few coefficients to represent curves. In wavelet transform, the decomposition only use squares at every direction or scale, while the local anisotropic transforms use rectangles with different sizes and directions. The figure 1 demonstrates the difference between wavelet transform and global directional multi-scale decomposition called local directional multi-scale curvelet-like.



*Fig. 2.* Wavelet transform uses squares, while the local anisotropic transforms use rectangles with different sizes and directions in image decomposition

Primarily, the first form of curvelet transform is based on multi-scale filters and orthogonal ridgelet transform in the unite blocks of the partition at each scale. The second form of curvelet transform is introduced in [5] with main properties: does not based on orthogonal; only use three-parameter vector to represent curvelet coefficients. The curvelet form II is developed in discrete domain with fast algorithm called FDCT (Fast discrete curvelet transform) [2].

Given  $\mu = (j, k, l)$ , in which j = 0, 1, ... is decomposition level,  $l = 0, 1, ..., 2^{\lfloor j/2 \rfloor} - 1$  represents angle index and  $k = (k_1, k_2)$  represents the position in 2D space. The algorithm is summarized in Algorithm 1.

Algorithm 1. The algorithm of fast discrete curvelet transform form II

S1. Convert the original data to Fourier domain.

S2. Localize the relationship between decomposition level and angle index. The angle at level *j* and angle index *l* is determined by  $\theta_{(j,l)} = 2\pi . 2^{-\lfloor j/2 \rfloor} . l$ .

S3. The diagonal matrix is defined by  $D_j = \begin{pmatrix} 2^j & 0 \\ 0 & 2^{\lfloor j, \alpha \rfloor} \end{pmatrix}$ . The value  $\alpha$  is defined

$$by, \alpha = \begin{cases} 1 & wavelet \\ 0 & ridgelet \\ 1/2 & curvelet \end{cases}$$

- S4. The location:  $k_{\delta} = (k_1 \cdot \delta_1, k_2 \cdot \delta_2)$ , with  $\delta_1 = 14/3$ ;  $\delta_2 = 10\pi/9$ .
- S5. Determine curvelet coefficients by formula (1)

$$\gamma_{\mu}(x) = 2^{3j/4} \gamma_{(j)}(D_{j}R_{\theta_{(j,l)}}x - k_{\delta})$$
(1)

In which,  $\gamma_{(j)}$  can be defined by:  $\gamma(x_1, x_2) = \psi(x_1)\varphi(x_2)$ ,  $\varphi$  is any scale filter,  $\psi$  is wavelet filter. For example, some scale filters  $\varphi$  can be

Daubechies 7-9	High: (0.037828, -0.023849, -0.11062, 0.3774, 0.8527, 0.3774, -0.11062, -0.023849, 0.037828)
	Low: (-0.064539, -0.040689, 0.41809, 0.78849, 0.41809, -0.040689 -0.064539)
Daubechies-4	High: (-0.23038, 0.71485, -0.63088, -0.027984, 0.18703, 0.030841 -0.032883, -0.010597)
	Low: (-0.010597, 0.032883, 0.030841, -0.18703, -0.027984 0.63088, 0.71485, 0.23038)
Burt	High: (-0.070711, 0.35355, 0.84853, 0.35355, -0.070711)
	Low: (-0.015152, -0.075761, 0.36871, 0.85863, 0.36871, -0.075761, -0.015152)

Table 1. Some popular filters used in wavelet transform

In original data, each wedge is applied to a part of data that has been sketched by the matrix  $D_j$ , and limitation of angle width is  $2^{j/2}$ , and they are normalized to [0,1] domain, therefore the size of wedge satisfies: length  $\approx 2^{-j/2}$  (by orientation  $\theta_{(j,l)}$ ), width  $\approx 2^{-j}$ . Every wedge satisfies the anisotropic rule, more particularly it satisfies parabolic partition property: width  $\approx$  length<sup>2</sup>.

In Fourier domain, the curvelet coefficients have compact support and  $\hat{\gamma}_{\mu}$  is localized for the wedge which is determined by:

$$\left| \pm \xi, \quad 2^{j} \leq \left| \xi \right| \leq 2^{j+1}, \left| \theta - \theta_{j,l} \right| \leq \pi \cdot 2^{-\lfloor j/2 \rfloor} \right\}$$

$$\tag{2}$$

That means, the curvelet coefficients has a support in the domain of length  $2^{j}$ , and width  $2^{j/2}$ . This shows that, at the decomposition scale  $2^{-j}$ , closure compound of

curvelet coefficients is in shape of thin trip with length  $2^{j/2}$  and width  $2^j$  (figure 3). The details of curvelet form II are introduced in [3].



Fig. 3. The partition of curvelet coefficients in frequency domain

The curvelet coefficients of 2D data are arranged in wedges in coronas (figure 3). Observing that, if there is data existing in the partition window, then there are significant curvelet coefficients in the correspondence wedge (the white color in figure 4). This is a base that we use crude curvelet coefficients or together with Zernike moments for SLF classification.



Fig. 4. The original microtubules image and curvelet form II coefficients with 03 decomposition levels using wavelet Db4

### 3. The Zernike moments

The next feature style for SLF classification is **Zernike** moment. These features are considered as orthogonal moments and can be used to represent image. The Zernike features are rotation invariant and can be built through polynomial functions. In particular, Zernike polynomials can be used to represent grey-level contribution. The Zernike polynomial order *n* with *m* repeats in image f(x,y) is determined by

$$V_{nm}(x, y) = V_{nm}(\rho, \theta) = R_{n,m}(\rho)e^{jm\theta}, \ \rho = \sqrt{x^2 + y^2}$$
(3)

|m| < n, n - |m| is even,  $0 \le l \le n$ ,  $\theta = \tan^{-1}(y/x)$ .  $R_{nm}$  is Zernike polynomial in polar coordinate system

$$R_{nm}(x,y) = \sum_{s=0}^{(n-|m|)/2} (-1)^m \frac{(n-s)!}{s!((n-2s+|m|)/2)!((n-2s-m)/2)!} (x^2 + y^2)$$
(4)

 $V_{nm}$  is a set of complex polynomials and determine the orthogonal base in  $x^2 + y^2 \le 1$ . Zernike moment are determined by

$$Z_{nm} = \frac{n+1}{\pi} \sum_{x} \sum_{y} f(x, y) V_{nm}^{*}(x, y), \quad x^{2} + y^{2} \le 1$$
(5)

 $V_{nm}^{*}(x, y) = V_{n,-m}(x, y)$ 



Fig. 5. The original image and its Zernike moments degree 9

# 4. Combination of curvelet transform and Zernike moments

In this paper, we suggest the combination between curvelet-like coefficients and Zernike moment to build the feature vectors in recognition. The main idea is that we apply the Zernike moment in the corona of curvelet-like transform to get multi-corona Zernike features. More specially, instead of using uniform rotation as in original Zernike moments for the angles which is determined by (3), we use the same directions of the wedges at different levels. Beside it, lengths of vectors  $\rho$  in (3) is limited by the size of corona in curvelet-like transform at different decomposition levels. In this case, the original image is decomposed to the dyadic coronas. Then, the Zernike moments are determined in those coronas instead of the whole image. The distinguishing point for Zernike moments in this case is that we use the radius  $\rho$  limited for image region in corona instead of using  $x^2 + y^2 \le 1$  as in original Zernike. The rotations in Zernike calculations are a rotation angle of the wedges in the curvelet-like transform at specific decomposition level. This is a merit point in our method because the curvelet-like transforms have already determined optimized cover grid. Therefore, we do not need to use uniform grid as in original Zernike moments. Figure 6 demonstrates the Zernike moments on the corona.



Fig. 6. Schema of Zernike moments in the coronas of curvelet-like transform

In particular, if we use the curvelet transform, then Zernike orders of *n* and *m* are repeated in the curvelet coefficients  $\alpha_{\mu}$  of specific corona is determined as follows:

$$V_{nm}(\alpha_{\mu}) = R_{n,m}(\gamma_{\mu})e^{i.m\theta}$$

$$Z_{nm} = \frac{n+1}{\pi} \sum_{\alpha_{\mu}} \gamma_{\mu} V_{nm}^{*}(\gamma_{\mu}), ||k|| \le 1$$
(8)

Where,  $\mu = (j,k,l)$  if using the 2<sup>nd</sup> form of the curvelet, and  $||k|| \le 1$ ,  $\alpha_{\mu}$  is determined by formula (1). The rotation  $\theta$  is fixed at the specific level *j* and the angle index *l*. That means, with the wedge determined by (j,l), the Zernike polynomial is calculated in the curvelet coefficients such that

$$\theta = \theta_{(j,l)} = 2\pi . 2^{-\lfloor j/2 \rfloor} l, \ l = 0, 1, \dots, 2^{\lfloor j/2 \rfloor} - 1$$
(9)

### 5. Experiment results

We have experimented in 2D fluorescence image sets which are introduced in [7], [8] with.

Description	Quantity	Sample image
The dataset of SLF images in experiment.	2598	

The images represent HELA or CHO cells in low resolution. There are from 73 to 98 images in a class. The image sets are extracted from the protein research projects in [6]. There are 2598 images in experimental set. A number of trained samples occupies 25-30% in experiment set and has been chosen randomly many times in sample set. A number of Zernike moments of each corona is used with Zernike polynomial degree 9, that means, n = 9, m = (0..9) in (5), with l = 4,6,8 represents the minimum degree of polynomial. Table 2 represents a number of features of Zernike moment vector. We use 45 features for each corona of curvelet transform. In the coarsest level, we use standard Zernike moments as usual. It is because the size of data at this level is small to partition.

No	Value <i>l</i>	Value <i>n</i>	Value <i>m</i>	Number of Zernike moments
1	4	4	0,1,2,3,4	45
		5	0,1,2,3,4,5	
		6	0,1,2,3,4,5,6	
		7	0,1,2,3,5,5,6,7	
		8	0,1,2,3,4,5,6,7,8	
		9	0,1,2,3,4,5,6,7,8,9	
2	6	6	0,1,2,3,4,5,6	34
		7	0,1,2,3,5,5,6,7	
		8	0,1,2,3,4,5,6,7,8	
		9	0,1,2,3,4,5,6,7,8,9	
3	8	8	0,1,2,3,4,5,6,7,8	19
		9	0,1,2,3,4,5,6,7,8,9	

Table 2. A number of Zernike moments at different degrees

Table 3. The experiment result of pattern recognition on the sub-cellular image sets (08 classes) based on multi-coronas curvelet Zernike moments and Zernike moments with SVM

Sub-cellular	Multi- coronas Zernike moments on curvelet-like	Zernike moments
filamentous form of the cytoskeletal protein actin	89	81
endosomal protein transferring receptor	64	67
endoplasmic reticulum	75	75
Golgi protein giantin	71	70
Golgi protein GPP130	71	68
lysosomal protein ERDAK	62	69
cytoskeletal protein tubulin	72	71
mitochondrial protein	75	73

The experimental results of our method are worse than normal Zernike in two types of class, and better than in the rest classes. It is because in these classes, most patterns are in black. The true patterns (white color) are small. Therefore, the curveletlike coronas cannot bring the good feature. However, in most classes with the bigger and complicated patterns, our suggestion is really better.

# 6. Conclusion

The curvelet-like transforms can contribute in many fields and show the merit points in comparison to wavelet. The development of feature set that is based on the coefficients of curvelet-like transform is very suitable to classify SLF images. In this paper, we suggest the outstanding solution with combination Zernike moments on the corona of curvelet-like coefficients. The experimental results show the better recognition in comparing to Zernike moments. We believe that our solution can achieve the better result when we integrate some basic features such as Haralick, geometry, and morphology, and the disadvantage with black images can be passed.

#### REFERENCES

- 1. Eero P Simoncelli, William T Freeman (1995), "The steerable pyramid: a flexible architecture for multi-scale derivative computation", 2nd Annual IEEE International Conference on Image-Processing. Washington, DC. October.
- 2. Emmanuel Candès. Laurent Demanet, David Donoho, Lexing Ying (2006), "Fast Discrete Curvelet Transform", Stanford University.
- 3. Emmanuel J. Candes, David L. Donoho (2000), "Curvelets, MultiResolution Representation and Scaling Laws", Stanford University Press USA.
- 4. Emmanuel J. Candes, David L. Donoho (2000), "Curvelets: A surprisingly Effective Non-Adaptive Representation for objects with edges", Vanderbilt University Press, Nashville, TN, USA.
- 5. F.G. Mayer, A. Z. Averbuch, J.-O. Stromberg (2000), "Fast adaptive wavelet packet image compression," *IEEE Trans. Image Processing*, vol 9, no 5, pp. 792-800.
- 6. Gene Kim, MyungHo Kim (2001), "2D Electrophoresis Gel Image and Diagnosis of disease", Bioinformatics Frontier Inc. 93 B Taylor Ave East Brunswick, NJ.
- M. Velliste, R. F. Murphy (2002), "Automated Determination of Protein Sub-cellular Locations from 3D Fluorescence Microscope Images," *Proc2002 IEEE Intl Symp Biomedl Imag*, pp. 867-870.
- 8. Michael V. Boland, Robert F. Murphy (2001), "A neural network classifier capable of recognizing the pattern of all subcelluars structures in fluorescence microscopy images of HeLa cells", Bioinformatics vol 17, no 12.

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