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# ON THE EXISTENCE OF SOLUTIONS FOR VECTOR QUASIEQUILIBRIUM PROBLEMS

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**ABSTRACT**

*In this paper, we establish some existence theorems for vector quasiequilibrium problems in real locally convex Hausdorff topological vector spaces by using Kakutani-Fan-Glicksberg fixed-point theorem. Moreover, we also discuss the closedness of the solution sets for these problems. The results presented in the paper are new and improve some main results in the literature.*

***Keywords:*** vector quasiequilibrium problems, Kakutani-Fan-Glicksberg fixed-point theorem, closedness.

**TÓM TẮT**

***Sự tồn tại nghiệm cho bài toán tựa cân bằng vectơ***

*Trong bài báo này, chúng tôi thiết lập một số định lí tồn tại nghiệm cho bài toán tựa cân bằng vectơ trong không gian tôpô Hausdorff thực lồi địa phương bằng cách sử dụng định lí điểm bất động Kakutani-Fan-Glicksberg. Ngoài ra, chúng tôi cũng thảo luận tính đóng của các tập nghiệm của bài toán này. Kết quả trong bài báo là mới và cải thiện một số kết quả chính trong tài liệu tham khảo.*

***Từ khóa:*** các bài toán tựa cân bằng vectơ, định lí điểm bất động Kakutani-Fan-Glicksberg, tính đóng.

## 1. Introduction

The equilibrium problem was named by Blum and Oettli [2] as a generalization of the variational inequality and optimization problems. This model has been proved to contain also other important problems related to optimization, namely, optimization problems, Nash equilibrium, fixed-point and coincidence-point problems, traffic network problems, etc. During the last two decades, there have been many papers devoted to equilibrium and related problems. The most important topic is the existence conditions for this class of problems (see, e.g., [3-5], and the references therein).

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In 2008, Long et al. [7] introduced generalized strong vector quasi-equilibrium problems (for short, (GSVQEP)). Let X, Y and $Z$ be real locally convex Hausdorff topological vector spaces, $A \subset X$ and $B \subset Y$ are nonempty compact convex subsets, and $C \subset Z$ is a nonempty closed convex cone, and let $S: A \to 2^A, T: A \to 2^B$, $F: A \times B \times A \to 2^Z$ be set-valued mappings.

(GSVQEP): Find $\overline{x} \in A$ and $\overline{y} \in T(\overline{x})$ such that $\overline{x} \in S(\overline{x})$ and

$$F(\overline{x}, \overline{y}, x) \subset C, \forall x \in S(\overline{x}),$$

where $\overline{x}$ is a strong solution of (GSVQEP).

Very recently, Yang and Pu [9] established the system of strong vector quasi-equilibrium problems in locally convex Hausdorff topological vector spaces and discussed some existence results and stability of solutions for these problems. Motivated by research works mentioned above, in this paper, we introduce two the generalized quasiequilibrium problems in real locally convex Hausdorff topological vector spaces. We also establish existence conditions for these problems. Our results improve and extend from main results of Long et al in [7] and Yang-Pu in [9]. Let X, Y, Z be real locally convex Hausdorff topological vector spaces, $A \subseteq X$ and $B \subseteq Y$ are nonempty compact convex subset and $C \subseteq Z$ is a nonempty closed convex cone. Let $K_1: A \to 2^A$, $K_2: A \to 2^A$, $T: A \to 2^B$ and $F: A \times B \times A \to 2^Z$ be multifunctions. We consider the following generalized quasiequilibrium problems (in short, (QVEP$_1$) and (QVEP$_2$)), respectively.

(QVEP$_1$): Find $\overline{x} \in A$ such that $\overline{x} \in K_1(\overline{x})$ and $\exists \overline{z} \in T(\overline{x})$ satisfying

$$F(\overline{x}, \overline{z}, y) \cap C \neq \emptyset, \forall y \in K_2(\overline{x})$$

and

(QVEP$_2$): Find $\overline{x} \in A$ such that $\overline{x} \in K_1(\overline{x})$ and $\exists \overline{z} \in T(\overline{x})$ satisfying

$$F(\overline{x}, \overline{z}, y) \subset C, \forall y \in K_2(\overline{x}).$$

We denote that $S_1(F)$ and $S_2(F)$ are the solution sets of (QVEP$_1$) and (QVEP$_2$), respectively.

The structure of our paper is as follows. In the remaining part of this section we recall definitions for later uses. Section 3, we establish some existence theorems by using Kakutani-Fan-Glicksberg fixed-point theorem for vector quasiequilibrium problems with set-valued mappings in real locally convex Hausdorff topological vector spaces.

## 2. Preliminaries

In this section, we recall some basic definitions and their some properties.

***Definition 2.1.*** ( [1]) Let X, Y be two topological vector spaces and $A$ a nonempty subset of $X$ and let $F: A \to 2^Y$ be a set-valued mappings, with $C \subset Y$ is a nonempty closed compact convex cone.

(i) $F$ is said to be *lower semicontinuous (lsc) at* $x_0 \in A$ if $F(x_0) \cap U \neq \emptyset$ for some open set $U \subseteq Y$ implies the existence of a neighborhood $N$ of $x_0$ such that $F(x) \cap U \neq \emptyset, \forall x \in N$. $F$ is said to be lower semicontinuous in $A$ if it is lower semicontinuous at all $x_0 \in A$.

(ii) $F$ is said to be *upper semicontinuous (usc) at* $x_0 \in A$ if for each open set $U \supseteq G(x_0)$, there is a neighborhood $N$ of $x_0$ such that $U \supseteq F(x), \forall x \in N$. $F$ is said to be upper semicontinuous in $A$ if it is upper semicontinuous at all $x_0 \in A$.

(iii) $F$ is said to be *continuous in* $A$ if it is both lsc and usc in $A$.

(iv) $F$ is said to be closed at $x_0$ if and only if $\forall x_n \to x_0, \forall y_n \to y_0$ such that $y_n \in F(x_n)$, we have $y_0 \in F(x_0)$.

***Definition 2.2.*** ( [1]) Let X, Y be two topological vector spaces and $A$ a nonempty subset of $X$ and let $F: A \to 2^Y$ be a set-valued mappings, with $C \subset Y$ is a nonempty closed compact convex cone.

(i) $F$ is called *upper $C$-continuous at* $x_0 \in A$, if for any neighbourhood $U$ of the origin in $Y$, there is a neighbourhood $V$ of $x_0$ such that, for all $x \in V$,

$$F(x) \subseteq F(x_0) + U + C, \forall x \in V.$$

(ii) F is called *lower $C$-continuous at* $x_0 \in A$, if for any neighbourhood $U$ of the origin in $Y$, there is a neighbourhood $V$ of $x_0$ such that, for all $x \in V$,

$$F(x_0) \subseteq F(x) + U - C, \forall x \in V.$$

***Definition 2.3.*** ( [1]) Let $X$ and $Y$ be two topological vector spaces and $A$ be a nonempty convex subset of $X$. A set-valued mapping $F: A \to 2^Y$ is said to be $C$-*convex* if for any $x, y \in A$ and $t \in [0,1]$, one has

F(tx+(1-t)y) $\subseteq$ tF(x)+(1-t)F(y)-C.

F is said to be $C$-*concave* is -$F$ is $C$-convex.

***Definition 2.4.*** *(* [1]) Let $X$ and $Y$ be two topological vector spaces and $A$ be a nonempty convex subset of $X$. A set-valued mapping $F: A \to 2^Y$ is said to be properly $C$-*quasiconvex* if for any $x, y \in A$ and $t \in [0,1]$, we have

either F(x) $\subseteq$F(tx+(1-t)y)+C,

or F(y) $\subseteq$F(tx+(1-t)y)+C.

**Lemma 2.1.** ([8]) *Let $X$ and $Z$ be two Hausdorff topological spaces and $A$ a onempty subset of $X$ and $F: A \to 2^Z$ be a multifunction.*

*(i) If $F$ is upper semicontinuous at $x_0 \in A$ with closed values, then $F$ is closed at $x_0 \in A$;*

*(ii) If $F$ is closed at $x_0 \in A$ and $F(X)$ is compact, then $F$ is upper semicontinuous at $x_0 \in A$;*

*(iii) If $F$ has compact values, then $F$ is usc at $x_0$ if and only if for each net $\{x_\alpha\} \subseteq A$ which converges to $x_0$ and for each net $\{y_\alpha\} \subseteq F(x_\alpha)$, there are $y \in F(x)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \to y$.*

**Lemma 2.2.** (Kakutani-Fan-Glickcberg ([6])). *Let $A$ be a nonempty compact subset of a locally convex Hausdorff vector topological space $Y$. If $M: A \to 2^A$ is upper semicontinuous and for any $x \in A, M(x)$ is nonempty, convex and closed, then there exists an $x^* \in A$ such that $x^* \in M(x^*)$.*

## 3. Main Results

In this section, we discuss existence conditions and closedness of the solutions of vector quasiequilibrium problems by using Kakutani-Fan-Glicksberg fixed-point theorem.

**Theorem 3.1.** *Let X, Y, Z be real locally convex Hausdorff topological vector spaces, $A \subseteq X$ and $B \subseteq Y$ be nonempty compact convex subsets and $C \subseteq Z$ be a nonempty closed convex cone. Let $K_1: A \to 2^A$ is upper semicontinuous in $A$ with nonempty convex closed values, $K_2: A \to 2^A$ is lower semicontinuous in $A$ with nonempty closed values, $T: A \to 2^B$ is upper semicontinuous in $A$ with nonempty convex compact values. Let $F: A \times B \times A \to 2^Z$ be a set-valued mapping satisfy the following conditions:*

*(i) for all* $(x,z)\in A\times B$, $F(x,z,K_2(x))\cap C\neq\emptyset$;

*(ii) for all* $(x,z)\in A\times B$, *the set* $\{a\in K_1(x): F(a,z,y)\cap C\neq\emptyset, \forall y\in K_2(x)\}$ *is convex;*

*(iii) the set* $\{(x,z,y)\in A\times B\times A: F(x,z,y)\cap C\neq\emptyset\}$ *is closed.*

*Then, the* (QVEP$_1$) *has a solution, i.e., there exists* $\bar{x}\in A$ *such that* $\bar{x}\in K_1(\bar{x})$ *and* $\exists\bar{z}\in T(\bar{x})$ *satisfying* $F(\bar{x},\bar{z},y)\cap C\neq\emptyset, \forall y\in K_2(\bar{x})$.

*Moreover, the solution set of the (QVEP$_1$) is closed.*

***Proof.*** For all $(x,z)\in A\times B$, define a set-valued mapping: $\Phi: A\times B\to 2^A$ by

$$\Phi(x,z)=\{t\in K_1(x): F(t,z,y)\cap C\neq\emptyset, \forall y\in K_2(x)\}.$$

**I**. Show that $\Phi(x,z)$ is nonempty and convex.

Indeed, for all $(x,z)\in A\times B$, $K_1(x), K_2(x)$ are nonempty. Thus, by assumption (i), we have $\Phi(x,z)\neq\emptyset$. On the other hand, by the condition (ii), we also have $\Phi(x,z)$ is convex subset of $A$.

**II.** Show that $\Phi$ is upper semicontinuous in $A\times B$.

Since $A$ is compact, we need only show that $\Phi$ is a closed mapping. Indeed, Let a net $\{(x_\alpha,z_\alpha)\}\subseteq A\times B$ such that $(x_\alpha,z_\alpha)\to(x,z)\in A\times B$, and let $t_\alpha\in\Phi(x_\alpha,z_\alpha)$ such that $t_\alpha\to t_0$. We now need to show that $t_0\in\Phi(x,z)$. Since $t_\alpha\in K_1(x_\alpha)$ and $K_1$ is upper semicontinuous with nonempty closed values. Hence $K_1$ is closed, thus we have $t_0\in K_1(x)$. Suppose to the contrary $t_0\notin\Psi(x,z)$. Then, there exists $y_0\in K_2(x)$ such that

$$F(t_0,z,y_0)\cap C=\emptyset. \tag{3.1}$$

By the lower semicontinuity of $K_2$, there is a net $\{y_\alpha\}$ such that $y_\alpha\in K_2(x_\alpha)$, $y_\alpha\to y_0$. Since $t_\alpha\in\Phi(x_\alpha,z_\alpha)$, we have

$$F_1(t_\alpha,z_\alpha,y_\alpha)\cap C\neq\emptyset. \tag{3.2}$$

By the condition (iii) and (3.2), we have

$$F_1(a_0,z,y_0)\cap C\neq\emptyset. \tag{3.3}$$

This is a contradiction between (3.1) and (3.3). Thus, $t_0 \in \Phi(x,z)$. Hence, $\Phi$ is upper semicontinuous in $A \times B$.

**III.** Now we need to the solutions set $S_1(F) \neq \emptyset$.

Define the set-valued mapping $H : A \times B :\to 2^{A\times B}$ by

$H(x,z) = (\Phi(x,z), T(x)), \forall (x,z) \in A \times B.$

Then $H$ is upper semicontinuous and $\forall (x,z) \in A \times B, H(x,z)$ is a nonempty closed convex subset of $A \times B$. By Lemma 1.2, there exists a point $(x^*, z^*) \in A \times B$ such that $(x^*, z^*) \in H(x^*, z^*)$, that is

$x^* \in \Phi(x^*, z^*),\ z^* \in T(x^*),$

which implies that there exist $x^* \in A$ and $z^* \in T(x^*)$ such that $x^* \in K_1(x^*)$ and $F(x^*, z^*, y) \cap C \neq \emptyset$, i.e., $x^* \in S_1(F)$.

**IV.** Now we prove that $S_1(F)$ is closed. Indeed, let a net $\{x_\alpha, \alpha \in I\} \in S_1(F)$: $x_\alpha \to x_0$. As $x_\alpha \in S_1(F)$, there exists $z_\alpha \in T(x_\alpha)$ such that

$F(x_\alpha, z_\alpha, y) \cap C \neq \emptyset, \forall y \in K_2(x_\alpha).$

Since $K_1$ is upper semicontinuous with nonempty closed values. Hence $K_1$ is closed. Thus, $x_0 \in K_1(x_0)$. Since $T$ is upper semicontinuous with nonempty compact values. Thus $T$ is closed, hence we have $z \in T(x_0)$ such that $z_\alpha \to z$. By the condition (iii), we have

$F(x_0, z, y) \cap C \neq \emptyset, \forall y \in K_2(x_0).$

This means that $x_0 \in S_1(F)$. Thus $S_1(F)$ is closed. □

Passing to the problem (QVEP$_2$), we also have the following similar results as that of Theorem 3.1.

***Theorem 3.2.*** *Let X, Y, Z be real locally convex Hausdorff topological vector spaces, $A \subseteq X$ and $B \subseteq Y$ be nonempty compact convex subsets and $C \subseteq Z$ be a nonempty closed convex cone. Let $K_1 : A \to 2^A$ is upper semicontinuous in $A$ with nonempty convex closed values, $K_2 : A \to 2^A$ is lower semicontinuous in $A$ with nonempty closed values, $T : A \to 2^B$ is upper semicontinuous in $A$ with nonempty convex compact values. Let $F : A \times B \times A \to 2^Z$ be a set-valued mapping satisfy the following conditions:*

*(i) for all* $(x,z) \in A \times B$, $F(x,z,K_2(x)) \subset C$;

*(ii) for all* $(x,z) \in A \times B$, *the set* $\{a \in K_1(x) : F(a,z,y) \subset C, \forall y \in K_2(x)\}$ *is convex;*

*(iii) the set* $\{(x,z,y) \in A \times B \times A : F(x,z,y) \subset C\}$ *is closed.*

*Then, the* (QVEP $_2$) *has a solution, i.e., there exists* $\bar{x} \in A$ *such that* $\bar{x} \in K_1(\bar{x})$ *and* $\exists \bar{z} \in T(\bar{x})$ *satisfying* $F(\bar{x},\bar{z},y) \subset C, \forall y \in K_2(\bar{x})$.

*Moreover, the solution set of the (QVEP* $_2$*) is closed.*

***Proof.*** We omit the proof since the technique is similar as that for Theorem 3.1 with suitable modifications. □

If $K_1 = K_2 = K$, then (QVEP $_2$) becomes strong vector qusiequilibrium problem (in short, (SQVEP)), this problem has been studied in [7].

(SQVEP): Find $\bar{x} \in A$ and $\bar{z} \in T(\bar{x})$ such that $\bar{x} \in K(\bar{x})$ and

$F(\bar{x},\bar{z},y) \subset C,$ for all $y \in K(\bar{x})$.

Then, we have the following Corollary.

***Corollary 3.1.*** *Let X, Y, Z be real locally convex Hausdorff topological vector spaces,* $A \subseteq X$ *and* $B \subseteq Y$ *be nonempty compact convex subsets and* $C \subseteq Z$ *be a nonempty closed convex cone. Let* $K : A \to 2^A$ *is continuous in* $A$ *with nonempty closed convex values,* $T : A \to 2^B$ *is upper semicontinuous in* $A$ *with nonempty convex compact values. Let* $F : A \times B \times A \to 2^Z$ *be a set-valued mapping satisfy the following conditions:*

*(i) for all* $(x,z) \in A \times B$, $F(x,z,K(x)) \subset C$;

*(ii) for all* $(x,z) \in A \times B$, *the set* $\{a \in K(x) : F(a,z,y) \subset C, \forall y \in K(x)\}$ *is convex;*

*(iii) he set* $\{(x,z,y) \in A \times B \times A : F(x,z,y) \subset C\}$ *is closed.*

*Then, the (SQVEP) has a solution, i.e.,, there exists* $\bar{x} \in A$ *such that* $\bar{x} \in K(\bar{x})$ *and*

$\exists \bar{z} \in T(\bar{x})$ *satisfying* $F(\bar{x},\bar{z},y) \subset C, \forall y \in K(\bar{x})$.

*Moreover, the solution set of the (SQVEP) is closed.*

If $K_1(x) = K_2(x) = K(x), T(x) = \{z\}$ for each $\bar{x} \in A$, then (QVEP $_2$) becomes strong vector equilibrium problem (in short,(SVEP)), this problem has been studied in [9].

**Corollary 3.2.** *Let X, Y, Z be real locally convex Hausdorff topological vector spaces,* $A \subseteq X$ *and* $B \subseteq Y$ *be nonempty compact convex subsets and* $C \subseteq Z$ *be a nonempty closed convex cone. Let* $K : A \to 2^A$ *is continuous in* $A$ *with nonempty closed convex values. Let* $F : A \times B \times A \to 2^Z$ *be a set-valued mapping satisfy the following conditions:*

*(i) for all* $(x,z) \in A \times B$, $F(x, K(x)) \subset C$;

*(ii) for all* $x \in A$*, the set* $\{a \in K(x) : F(a, y) \subset C, \forall y \in K(x)\}$ *is convex;*

*(iii) the set* $\{(x, y) \in A \times A : F(x, y) \subset C\}$ *is closed.*

*Then, the (SVEP) has a solution, i.e., there exists* $\bar{x} \in A$ *such that* $\bar{x} \in K(\bar{x})$ *satisfying* $F(\bar{x}, y) \subset C, \forall y \in K(\bar{x})$.

*Moreover, the solution set of the (SVEP) is closed.*

***Remark 3.1.*** In the special case as above, Corollary 3.1 and Corollary 3.2 reduce to Theorem 3.1 in [7] and Theorem 3.3 in [9], respectively. However, our Corollary 3.1 and Corollary 3.2 are stronger than Theorem 3.1 in [7] and Theorem 3.3 in [9]. Noting that, our Theorem 3.1 is new.

The following example shows that in this special case, all assumptions of Corollary 3.1 are satisfied. However, Theorem 3.1 in [7] is not fulfilled. The reason is that $F$ is not lower $(-C)$-continuous.

***Emxaple 3.1.*** Let $X = Y = Z = \mathbb{R}, A = B = [0,1], C = [0, +\infty)$ and let $K_1(x) = K_2(x) = [0,1]$ and $T_1(x) = T_2(x) = [\frac{1}{5}, 1]$

$$F(x,z,y) = F(x) = \begin{cases} [\frac{1}{3}, 1] & \text{if } x_0 = \frac{1}{3}, \\ [1,3] & \textit{otherwise.} \end{cases}$$

It is clear to see that all the assumptions of Corollary 3.1 are satisfied. So by this corollary the considered problem has a solution. However, $F$ is not lower $(-C)$-continuous at $x_0 = \frac{1}{3}$. Also, Theorem 3.1 in [7] does not work.

The following example shows that all the assumptions of Corollary 3.1 and Corollary 3.2 are satisfied. But, Theorem 3.1 in [7] and Theorem 3.3 in [9] are not fulfilled. The reason is that $F$ is not upper $C$-continuous.

***Emxaple 3.2.*** Let $X = Y = Z = \mathbb{R}$, $A = B = [0,1], C = [0,+\infty)$ and let $K_1(x) = K_2(x) = [0,1]$ and $T(x) = \{z\}$

$$F(x,z,y) = F(x) = \begin{cases} [1,\frac{3}{2}] & \text{if } x_0 = \frac{1}{3}, \\ [\frac{1}{6},\frac{2}{3}] & otherwise. \end{cases}$$

It is not hard to check that all the assumptions of Corollary 3.1 and Corollary 3.2 are satisfied. However, $F$ is not upper $C$-continuous at $x_0 = \frac{1}{3}$. Also, Theorem 3.1 in [7] and Theorem 3.3 in [9] do not work.

The following example shows that the all assumptions of Corollary 3.1 and Corollary 3.2 are satisfied. However, Theorem 3.1 in [7] and Theorem 3.3 in [9] are not fulfilled. The reason is that $F$ is not properly $C$-quasiconvex.

***Emxaple 3.3.*** Let $X = Y = Z = \mathbb{R}$, $A = B = [0,1], C = [0,+\infty)$ and let $K_1(x) = K_2(x) = [0,1]$ and $T(x) = \{z\}$

$$F(x,z,y) = F(x) = \begin{cases} [1,4] & \text{if } x_0 = \frac{1}{4}, \\ [\frac{1}{5},1] & otherwise. \end{cases}$$

It is easy to see that all the assumptions of of Corollary 3.1 and Corollary 3.2 are not fulfilled. However, $F$ is not properly $C$-quasiconvex at $x_0 = \frac{1}{4}$. Thus, it gives case where of Corollary 3.1 and Corollary 3.2 can be applied but Theorem 3.1 in [7] and Theorem 3.3 in [9] do not work.

- ***Conflict of Interest:*** *Authors have no conflict of interest to declare.*

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