



R. CARNAP'S VIEW ON *THE SEMANTIC SYSTEM* IN *INTRODUCTION TO SEMANTICS*

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ABSTRACT

Rudolf Carnap is a great philosopher of modern Western philosophy of the twentieth century. His thoughts relate to the topics of logics and linguistics in which the conception of semantics occupies an important position. He built a fairly complete semantic system in his classic work, Introduction to Semantics. The study of the logical thoughts of semantics of R. Carnap not only has important theoretical significance for philosophy and logics, but also suggests problems of linguistics and mathematics.

Keywords: logics, modern Western philosophy, linguistics.

1. What is the *semantic system*?

In his semantic work, *Introduction to semantics*, Carnap introduced his conception of *the semantic system*: “a system of rules, formulated in a metalanguage and referring to an object language, of such a kind that the rules determine a truth-condition for every sentence of the object language, i.e. sufficient and necessary conditions for its truth” (Carnap 1948, p.22). Thus, it can be seen *that his semantic system is a system of rules governing the use of object language*. When all cognition paradigms adhere to these criteria, the thinking process will operate properly and reveal the truth.

The distinction between *object language* and *metalanguage* is drawn from the fact that: If the *object language* contains a class of expressions, then the *meta-language* contains a class of rules for correct use of expressions in the object language. In other words, according to R. Carnap's view, *metalanguage is philosophy*.

According to Carnap, the rules were built in *pure semantics*: “On the other hand, we may set up a system of semantical rules, whether in close connection with a historically given language or freely invented; we call this a *semantical system*. The construction and analysis of semantical systems is called pure semantics” (Carnap 1948, p.11-12). Unlike the *pure semantics*, *descriptive semantics* is only concerned about describing *facts* and this is the kind of semantics used in specific sciences, creating the content of those sciences.

In *Introduction to semantics*, Carnap had developed *semantic systems* containing four main types of rules, “commanding” the reasoning process: (1) classification of symbols, (2) rules of formation, (3) rules of designation, and (4) rules of truth.

2. The structure of the semantic system

2.1. Classification of symbols

Logic is the science subject of which object research is the abstract thinking of human beings. It aims to establish basic principles and methods that are to build reasonable arguments, follow certain objective laws of thinking. Logic lays down two specific rules as two “distinct nature signs” that do not overlap with other disciplines, which are the rules and descriptions. The rule is that this science builds a system of goals-oriented principles that answer the question: “How do I argue?”. The description of logic “manifests” in separating its research object in the realm of thinking, not researching the physical mechanism or psychological mechanism of reasoning, but only simply describes how the argument should be formulated.

In history of logic, *symbolic logic* is considered a branch of logic with a wide range of applications and extremely powerful functions. This sub-discipline uses a formal language consisting of abstract symbols play role as a tool for effective thinking process. The whole logical positivism thought of R. Carnap was formed and developed in the direction of symbol logic, so he opined that the symbol is “the ultimate units of the expressions of languages” (Carnap 1948, p.4). On that basis, *classification of signs* was the first step that Carnap concerned about, and it is also a prerequisite for the next parts in semantic systems.

Carnap divided the symbol in its logical system into three basic categories, namely, individual constants of zero level predicates (which he later primarily used the term “predicator”) belong to first degree and logical connections. In *Introduction to Semantics*, Carnap built three basic semantic systems, namely S_3 , S_4 and S_6 of which first part was about the classification of signs. However, the classification of symbols in each system does not mean listing all symbols Carnap used in his symbolic logic, but he only selects symbols that were related to semantic content contained in the system, and this classification had to be in close cooperation with other rules in S .

When establishing S_3 , S_4 and S_6 , Carnap used *individual symbols* (including instance *variables*) in (‘a’, ‘b’, ‘c’), pr predicates (‘P’, ‘Q’), quantifier exist \exists and single symbols include logical connections \sim , \vee , \bullet , \supset , \equiv and (,). Besides, in S_6 , Carnap also included variable I (‘x’, ‘y’ ...) to represent infinite numbers. Carnap clearly stated the scope of using symbols to express semantic content which was: “We shall later apply these symbols chiefly in examples of sentences in object language, but occasionally also in a metalanguage.” (Carnap 1948, p.15)

Thus, although the Carnap’s symbolic system is quite rich, we only use those that best match the purpose and content of each system when building each specific semantic system. Similarly, the logical connections that Carnap used, in essence, are called *truth-functional operators* in propositional logic. *Propositional operators* are words or phrases

(or, and, if and only if...) that aggregate atomic propositions into compound propositions. *The truth function* is defined as the type of function based on the truth value of the atomic propositions to form the truth value of the compound proposition. The combination of *the proposition operator* and *the truth function* will create the *truth-functional operator* with the logical connections that play role as the “smallest bricks” to build the “castle” of propositional logic. However, Carnap did not present the specification of all the *truth-functional operators*. Before embarking on the construction of the S_3 semantic system, he only mentioned about two *truth-functional operators* \vee and \sim . However, instead of using the symbol of object language, Carnap used signs of metalanguage.

The *classification of signs* carried out by Carnap in the construction of the semantic system first of all, however, it is required close coordination with the following steps to create a complete system, ensuring that inference satisfy semantic rules to determine the truth value of sentences in metalanguage.

2.2. Rules of formation

The second part of the S semantic system of Carnap is *the formation rule*. The forming rule is a set of rules defining a sentence that is considered to be *standard* and *precise* in the syntax in S . In other words, the rule of formation provides a *recursive definition* of “true sentence in S ”. At the same time, it determines the different types of symbols mentioned in the *classification of signs* that can be combined with which syntax, to form valid sentences in a semantic system.

In the S_3 and S_4 system, when making the formation rule, Carnap constructed the following expressions A:

- | | | |
|-----------------------------|-----------------------------|--------------------------|
| (a) pr(in), | (b) $\sim(S_i)$, | (c) $(S_i) \vee (S_j)$ |
| (d) $(S_i) \bullet (S_j)$, | (e) $(S_i) \supset (S_j)$, | (f) $(S_i) \equiv (S_j)$ |

In S_6 system, he added the *functions* definition about *sentential functor* (A_κ) in S_6 and define the *sentence* (S) in S_6 .

It can be said that the rule of formulation is a set of rules that gives a tool to define an expression (Carnap used the term “sentence”) to be an expression in an S semantic system or not. If A_κ matches with the given rules, it is considered standard in S and *vice versa*, it is not a *well-formed formula* (*wff*) in S (i.e. *grammatically incorrect*). In symbolic logic, the rules of formation play a very important role because it is “for terms and for well-formed formulas” (Walicki 2016, p.31). Based on the rules of formation and the given kind of *well-formed formula*, we can see that Carnap had mixed expressions of propositional logic and predicate logic in the same semantic system and given the general rule for them without a clear separation.

The variables that Carnap used in his own logic and the construction of rules to be combined into sentences in S can be considered as *metalinguistic variables* (or can be briefly called *metavariables*). These are the *variables* that make up the *meta-language* and

used to provide propositions about expressions of *language-objects*. The difference between the *metavariable* and the usual *variable* is that it does not represent a proposition with the specific content of the object-language but represents all propositions of the object-language.

When building rules formed in S_6 , *sentential function* is defined by Carnap as follows: “An expression A_λ in S_6 is a *sentential function* in $S_6 \stackrel{\text{Def}}{=} A_\lambda$ has one of the following forms: $\alpha \text{ pr}(i)$, $\beta \sim(A_i)$, where A_i is a sentential function, $\gamma (A_i) \vee (A_j)$, where A_i and A_j are sentential functions containing the same variable.” (Carnap 1948, p.45). For explaining the *sentential function*, Carnap considered it in relation to free variables: “expression of sentential form with free variables.” (Carnap 1948, p.232). *The free variable* is determined through a correlation with *bound variable*: “A variable at a certain place in an expression is called **bound** if it stands at that place in an operator or in an operand whose operator contains the same variable; otherwise it is called **free**. An expression is called open, if it contains a free variable; otherwise **closed**.” (Carnap 1948, p.17)

So, it can be seen that, when building the definition of *sentential function* in S_6 and the concepts related thereto, Carnap used the kinds of *metavariable* of predicate logic, namely the *bound* and *free variables*. The determination of these two types of variables depended on the three types of **operators** that Carnap stated: the universal operator “ $(\forall x)$ ”, the existent operator “ $(\exists x)$ ” and the lambda operator “ (λx) ”. In addition, “...” is defined as **an operand** in each operator. In predicate logic, the determination of free and bound variables depends on the scope of activity of the two quantifiers - universal quantifier and existent quantifier (which Carnap called “operator”). The definition of *bound* and *free variables* can be generalized as follows: “An occurrence of a variable x in an expression A is said to be *bound* (or as a *bound variable*), if the occurrence is in a quantifier $\forall x$ or $\exists x$ or in the scope of a quantifier $\forall x$ or $\exists x$ (with the same x); otherwise, *free* (or as a *free variable*)” (Swart 2014, p.188). It can be seen that Carnap’s conception about the components of the symbolic logic given in the rules of formation were very close to the modern predicate logic. However, in this section, Carnap had not mentioned about the order of priority execution of operators and the main operator problem in a sentence.

2.3. Rules of designation

The rules of designation are the rules that determine designata to which the expressions in S refer. In other words, according to Carnap, rules of designation of semantic system S define the term “designation in S ” through the appearance of constants, variables and predicates present in S . These rules of designation do not assert or deny anything about the designata denoted by symbols, but they are merely treated as conventions for the use of symbols in a semantic system. In the S_3 semantic system, the rule firstly represents the relationship between an abstract symbol a and a reality u . Symbol a indicates a place and corresponding to each specific location u according to convention.

S_3 regulates “a” - Chicago, “b” - New York, “c” - Carmel. In addition to the rules of designation of entities, S_3 also establishes a rule for denoting properties with two predicates P and Q.

The rules of designation in the S_6 system also relate to other rules that are called *rules of determination* and *rules of value* by Carnap.

According to Carnap, *the rule of value* is intimately related to the rules of designation. It indicates **the value** of the variables used to represent the entity in the semantic system. The class of variables is considered as the **range of values** that is variable range over. Rules of value determine the range of values that any variable i gets in semantic system S . This range includes certain values that can refer to all things, phenomena, individuals and individuals that are in existence in a defined space-time period. Since the value of i is always referenced to an external object, it is called an instance in S .

If in S_3 , the rules merely indicate “a”, “b”, “c” to represent for Chicago, New York, and Carmel respectively, and set two properties of the three entities for “P” and “Q”, in the rules of value that appear in S_6 , variable i fluctuates in the range of values of cities in the US territory.

In combination with the rules of value, it is *rules of determination*. This type of rule **defines** entities with different types of *expressional functions* (containing free variables) whereas *sentential functions* are used to identify properties. In other words, this rule defines the term “define in S ”. In S_6 , a *sentential functions* A_k defined in S_6 attribute F when it meets all three conditions simultaneously:

- a. A_k has the form $pr_i(i_j)$ and pr_i designates F ,
- b. A_k is of the form $\sim(A_i)$, and F is the property of not having the property determined by A_i ,
- c. A_k has the form $(A_i) \vee (A_j)$ and F is the property of having either the property determined by A_i , or that determined by A_j or both.” (Carnap 1948, p.46)

Thus, according to Carnap, A_k is determined by: (1) The combination of a predicate which denotes a property with an infinite number i (plays as a variable) is one of the cities in the US territory (stated in the *rules of value*), and the predicate denotes attribute F ; (2) According to the rules of formation, $\beta \sim(A_i)$, attribute F is not defined by it; (3) the disjunction of $(A_i) \vee (A_j)$ and attribute F are defined in three cases: (I) A_i , (II) A_j , (III) A_i and A_j . However, Carnap himself realized that it was difficult to provide a definite rule if there were cases “ $(x)(\exists y)(.x..y..)$ ”. That is, according to the language of predicate logic, if the n -place [n -tuples] predicate have places more than one place, it is very difficult to determine A_k according to the above conditions.

If the rules of designation have a close relationship with the *rules of value* and the *rules of determination* because they are based on the inclusion and designation of variables in the S system, then the rules of designation can be deployed according to the relationship of denotation. In all three semantic systems S_3 , S_4 and S_6 , the rules of designation only apply to individual constants that are used to refer to cities in the US territory and the two predicates P and Q denote two attributes associated with corresponding landmarks.

However, according to Carnap, we can apply the relationship of designation to both the functors in S and this denotation follows the formula provided by Carnap which is $Des_S(u, v)$ of which shortened form is $Des(u, v)$ with the content that u designates v in S . The relationship expressed in a semantic system used by Carnap to denote the three types of designata are different individuals, attribute types and propositions. After such division, Carnap had re-structured the rules of designation in S .

Back to S_3 , the individuals are cities that will be represented by the functors $DesInd_{s_3}$ (in_i, x) simultaneously corresponds to the accompanying conditions of in_i then x will be referenced to a corresponding city. This relationship is built up corresponding to the properties in S_3 with functors $DesAttr_{s_3}$ (pr_i, F) and $DesProp_{s_3}$ (S_k, p).

It can be seen that between the *thinking* and the *being*, that is between the symbol and the object, there is an inseparable relationship. However, this relationship must be through an “intermediate stage” that is manifest, which can be expressed through the following diagram:



Therefore, the rules of expression posed by Carnap are seen as a bridge to serve the “operation” of logical expressions in his semantic system. However, if individual constants and predicates represent real objects or attributes as well as relationships, a sentence does not seem to be on behalf of any entity. So, whether using $DesProp_{s_3}$ in relation to the manifestation is fully satisfactory. This is just a problem and also an objection that Carnap himself raised in the process of building its semantic system.

2.4. Rules of truth

The *rules of truth* are the rule that indicates the necessary and sufficient conditions for the truth of each sentence in the S semantic system. In other words, the rules of truth will provide a definition of “true in S ”. When setting up the construction of the semantic system, Carnap argued that its mission was to interpret object language through determining the truth conditions of sentences in the system. To determine what is “true in S ”, it is necessary to first study the atomic propositions, then the molecular and compound propositions through logical connections.

Occasionally, when it comes to the problem of truth in mathematical logic, we often think about truth-values that are *true* and *false*. The truth-values in mathematical logic will be determined through constituent parts including propositions and function-truth operators (logical connection).

During the construction of the semantic system, Carnap established a framework of concepts that he called basic *concepts*. These are conceptions that are directly related to the concept of truth such as: true, false, implicate, equivalence, disjunction, exclusion, comprehensive inclusion. In order to serve the above work, Carnap had built new symbols and given definitions of general semantics and theorems.

First of all, Carnap first determined two *true* and *false* truth-values and then the definition and theorem of other basic concepts. The class of sentences K_i consists of partial sentences of this class. This class is considered to be the true question in the system S when all the parts of this are true in the S class. From the construction of definitions related to the concept of *true in S*, Carnap went to add a symbol to designate the entire sentence components in a class question and the whole class question, the symbol T . The concept *false* is defined by Carnap as follows:

“**D9-2.** T_i is **false** (in S) =_{Df} T_i belongs to S and is not true in S ” (Carnap 1948, p.35)

Thus, we can see that Carnap defined the concept *false* through concept *true* and based on the sentential class K of S . The following concepts such as *implicate*, *equivalent*, *disjunct*, *exclusive*, and *comprehensive* are defined based on two concepts of *true* and *false* and previous concepts:

“**D9-3.** T_j is an **implicate** of T_i (T_i implies T_j , $T_i \rightarrow T_j$) (in S) =_{Df} T_i and T_j belong to S , and either T_i is false or T_j is true (or both)” (Carnap 1948, p.36)

“**D9-4.** T_i **equivalent** to T_j (in S) =_{Df} T_i and T_j belong to S , and either both are true or neither of them is true” (Carnap 1948, p.36)

“**D9-5.** T_i is **disjunct** with T_j (in S) =_{Df} at least one of them is true (and hence, not both of them false”. (Carnap 1948, p.38)

“**D9-6.** T_i is **exclusive** of T_j (in S) =_{Df} not both of them are true (and hence, at least one is false)” (Carnap 1948, p.38)

“**D9-9.** T_i is *comprehensive* (in S) =_{Df} $T_i \rightarrow$ every sentence in S ” (Carnap 1948, p.40).

In addition to the two concepts of *true* and *false*, the concepts *implicate* and *equivalent* here act as the *material implicate* and *material equivalent*, not the logical ones. These two concepts will have distinctions with *logical implicate* and *logical equivalence* which will be presented in the *L-semantics* concerning *logical truths* in the *logical semantic* system of Carnap on *logical concepts* (or he called *L-concepts*). He also argued that the definition of his concepts of *implicate* and *equivalence* here is limited to the relationship between truth values between the two ranges of sentences and the sentential classes in the object language.

As can be seen that in the Carnap semantic system, these concepts are very important, but he focused on more logical concepts and endeavored a lot of effort to build them in his semantic system. Carnap's development of the concepts as mentioned above in S was just a step to create his logical concepts in logical semantics.

In all three semantic systems S_3 , S_4 , and S_6 , the rules of truth are set in relation to the classification of symbols, rules of formation and rules of designation.

In S_3 , the rules of truth is given based on the conditions related to the form of sentence S_k of which is a combination of a predicate \mathbf{pr}_i and an individual constant \mathbf{in}_j so that this combination is consistent with the existence of an entity with that attribute being outside objective reality, in particular they are two attributes of one in three US cities listed in S_3 . This can be considered to be the most important condition, and in a philosophical sense, it speaks of the requirement for unity between thinking and existence, between human perception and objectivity of things. Therefore, this is the prerequisite element and also the most important factor to reach the truth of the sentence S_k in the semantic system that Carnap had built. In addition to this condition, the rules of truth in S_3 also defines the truth-value of $\sim(S_k)$ and the truth-value of $(S_i) \vee (S_j)$ are the corresponding expressions in the rules of formation.

In S_4 , in addition to the rules of truth provided in S_3 , Carnap added other rules to correspond to the rules of formation, which is the rule of the value of conjunct and disjunct, implicate and equivalence between sentences in the system. In S_6 , due to the addition of rules of determination and rules of value, the rules of truth in S_6 in addition to inheriting the first three truth rules of S_3 Carnap also added rules of truth corresponding to i_j and A_k .

If the rules of designation use of the denoting relationship not only for individual constants and predicates but also for the functors, divide the designation for individuals, attributes, and propositions, using expressions $Des_s(u, v)$ as stated earlier, in this case, Carnap only listed a single rule of truth which is: " S_k is true in S_3 . $=_{Df}$ there is a (proposition) p such that $Des_s(S_k, p)$ and p ." (Carnap 1948, 51). Following this rule of truth, if we apply to variable $a = Chicago$, we will obtain the form " $Des_s('P(a)', Chicago$ is very wide)", and Carnap believed that based on the rules in S_3 , " $'P(a)'$ is true in S_3 when and only if Chicago is very wide." (Carnap 1948, p.51).

3. Conclusion

In conclusion, we can see that a semantic system according to the model that Carnap builds must have a structure consisting of four parts, namely:

1. Classification of symbols
2. Rules of formation
3. Rules of designation

4. Rules of truth

In the process of Carnap's construction and providing the symbols of S_3 , S_4 , and S_6 , it can be seen that he used the tools of mathematical logic and predicate logic thoroughly to build his semantic system. However, the semantic system that Carnap set up is aimed at a sub-semantic system that is included in this semantic system and is closely related to his logical positivism, which is *logical semantic system*.

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REFERENCES

- Carnap Rudolf. (1948). *Introduction to Semantics*. Cambridge: Harvard University Press.
 Swart Harrie de. (2018). *Philosophical and Mathematical logic*. Springer.
 Walicki Michal. (2016). *Introduction to mathematical logic*. Singapore: World Scientific Publishing.

**QUAN NIỆM CỦA R. CARNAP VỀ HỆ THỐNG NGŨ NGHĨA
 TRONG DẪN NHẬP VỀ NGŨ NGHĨA**

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TÓM TẮT

Rudolf Carnap là một nhà triết học vĩ đại của triết học phương Tây hiện đại thế kỉ XX. Tư tưởng của ông liên quan đến những chủ đề của logic học và ngôn ngữ học, trong đó quan niệm về ngữ nghĩa đóng một vai trò quan trọng. Ông đã xây dựng một hệ thống ngữ nghĩa tương đối hoàn chỉnh trong tác phẩm kinh điển của mình Introduction to Semantics. Việc nghiên cứu tư tưởng về ngữ nghĩa của R. Carnap không những có ý nghĩa lí luận to lớn đối với triết học và logic học, mà còn gợi mở nhiều vấn đề liên quan đến ngôn ngữ học và toán học.

Từ khóa: logic học, triết học phương Tây hiện đại, ngôn ngữ học.