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Research Article

DISTRIBUTIONS TRANSITION UNDER ORTHOGONAL RANDOM FLUCTUATIONS: AN APPLICATION TO SUPERCONDUCTIVITY-NORMAL PHASE TRANSITION

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ABSTRACT

It is well-known that some famous probability density functions (PDF) of random variables are associated with symmetries of these random variables. The Boltzmann and Gaussian PDFs that are invariant under translation and spherical transformations of their variables, respectively, are obvious and well-studied examples reflecting not only symmetries of many physical phenomena but also their underlying conservation laws. In physics and many other fields of interest of complexity, the transitions from the Boltzmann PDF to the Gaussian PDF, or at least from Boltzmann-like PDF to the Gaussian-like PDF, i.e from a sharp peak PDF to round peak PDF, are frequently observed. These observed phenomena might provide clues for a phase transition, namely second-order phase transition, where the symmetry of given physical quantities in the system under consideration is broken and changed to another one. The purpose of this work is to study this kind of transition in the superconductivity by investigating the transformation of envelope functions of electron and Cooper pair wavefunctions in spatial representation which might correspond to the change of symmetrical behavior of the space from its normal to superconducting states near the phase transition critical temperature.

Keywords: Superconductivity; The phase transition; Orthogonal Fluctuations

1. Introduction

Originally observed in nature and then essentially used as the fundamental example of collective behavior of a complex system where system properties would be investigated and "understood only from a holistic description of the properties of the entire system rather than from a reductionist description of individual" elements (Bak, 1996), the problem of sandpile dynamics is reformulated in the terms of probability theory and

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statistics that can map several similar phenomena in complex systems to single one that the shape of an initial probability distribution function or statistical density function of a given observable quantity concerning a random variable changes along with the evolution of time into the new one which is different from the initial. These phenomena appear quite often in many observable processes of complexity, from physical systems to social systems where the short-time data set provides a typical statistical weight function and the long time data set with the same variable provides another typical statistical weight function.

On the one hand, it is not easy to explicitly mechanically explain these phenomena due to the nature of the complexity of the system under consideration and uselessness of physically microscopic laws for the individuals within the system. However, in another hand, it is believed that there is at least a universal explanation for all kinds of transitions due to the universality of these phenomena. Many experimental and theoretical investigations, especially in econophysics focusing on various datasets of financial markets (Mantegna and Stanley, 1994, 1995, 1997; Bouchaud, 1999; Anh et al., 2013, 2014b,a, 2015, 2016), have been performed to find out a possible explanation providing deep insight for universality of the phenomenon and in theoretical aspect, the model of external fluctuations or noises originally introduced in (Anh et al., 2014a) would be a potentially promised approach.

The physical idea of the proposed approach is of the modeling all the external factors affecting on the given system by two kinds of effective fluctuations, in-space, and orthogonal ones. As it has been expected from physical reasoning, the in-space fluctuations would cause the expansion of initial probability distribution function to whole space, and fat and semi-fat tails of the final probability distribution function, while the orthogonal ones are associated to symmetry broken processes in the given system.

The purpose of this paper is twofold: first is to extend the formalism introduced in (Anh et al., 2014a) to consider the whole family of phenomena of probability distribution functions transition, including observable transitions and decay transitions of Gaussian probability distribution function. Second, by investigating the invariance of probability distribution functions, a kind of observable transitions is connected to the symmetry broken in superconductivity utilizing coordinate representation of wavefunctions in superconducting phase and normal phase to provide another insight into understanding the statistics of order parameters in a phase transition. The paper would be organized as follows. Sec. 2 introduces a general mathematical description for probability distribution functions transition under the influence of noises and their corresponding parameters. The probability distribution function transition under orthogonal noises is developed in Sec. 3.. The relationship between the probability distribution functions transition and superconducting phase transition is established in Sec. 4. And Sec. 5 contains some concluding remarks.

2. Mathematical Description of Fluctuation Contributions in Probability Distribution Functions

Provided a probability distribution function in form $f_i(x)$ such that

$$\int_{x_l}^{x_u} \mathrm{d}x f_i(x) = 1, \tag{1}$$

where the probability distribution function $f_i(x)$ is well-defined and continuous function determined in a finite interval $[x_l, x_u]$ of a random variable x. In most cases of present consideration, the interval $[x_l, x_u]$ is expanded in whole space ⌒, otherwise the integrand of eq. (1) can be rewritten as

$$f_i(x) = \begin{cases} 0 & (-\infty, x_l] \\ f_i(x) & [x_l, x_u] \\ 0 & [x_u, \infty) \end{cases} \tag{2}$$

to expand the integral to the whole space ⌒ of the random variable. However, it should be emphasized that the interval $[x_l, x_u]$ bounded by hard (close) or soft (open) boundaries is more interesting in identifying and analyzing asymptotical behaviors of final probability distribution function obtained by integrating over all additional degrees of freedom characterizing random fluctuations, such as the phenomenon of fat and semi-fat tails observed in complexity. Furthermore, due to the left-right symmetry of the most phenomena in complexity (Kleinert and Chen, 2007), the probability distribution function $f_i(x)$ could belong to a class of symmetric functions as

$$f_i(x) = f_i(|x - x_0|), \tag{3}$$

where x_0 is the symmetric point of the random variable x. And for the sake of simplicity, the point x_0 can be chosen as zero.

2.1. Contributions of Fluctuations on the Random Variable and its Distribution Functions

In general, it is impossible to explicitly take into account all the external factors affecting the system under consideration or on the dataset of a given observable of the system because of their unclear collective mechanism and also their complexity. However, it is still possible to model their effects in the terms of a random fluctuation consisted of two components, an in-space ζ and an orthogonal ε ones of which the mean value μ_i and standard variances σ_i would be associated with macroscopic effects of the fluctuations on the system.

The contributions of these fluctuations to variable should be written as

$$|x| \to \sqrt{(x-\zeta)^2 + \varepsilon^2}. \tag{4}$$

The expression eq. (4) implies that an orthogonal fluctuation ε should be considered as an additional dimension which extends $\frown$ to $\frown^2$, while in-space fluctuations ζ would make a shift of variable x within $\frown$ having the same dimensionality as a random variable x. Below, the physical significance of these two kinds of fluctuations will be discussed by associating them with changes from initial to final probability distribution functions.

Under the effects of fluctuations ε and ζ, the probability distribution function $f_i(x)$ will be changed, and a new distribution function $f_f(x)$ would be obtained by integrating over all fluctuation degrees of freedom, which are unseen in the process of transition. The observable final distribution function $f_f(x)$ should satisfy

$$f_f(x) = \mathcal{N}^{-1} \int_{C_\varepsilon} \mathrm{d}\varepsilon \int_{C_\zeta} \mathrm{d}\zeta\, g_{\text{orth}}(\varepsilon) g_{\text{in}}(\zeta) f_i\left(\sqrt{(x-\zeta)^2 + \varepsilon^2}\right), \tag{5}$$

where is $\mathcal{N}$ is some renormalized constant

$$\mathcal{N} = \int_{C_x} \mathrm{d}x \int_{C_\varepsilon} \mathrm{d}\varepsilon \int_{C_\zeta} \mathrm{d}\zeta\, g_{\text{orth}}(\varepsilon) g_{\text{in}}(\zeta) f_i\left(\sqrt{(x-\zeta)^2 + \varepsilon^2}\right), \tag{6}$$

and $g_{\text{in}}(\zeta)$ and $g_{\text{orth}}(\varepsilon)$ are probability distribution functions of in-space and orthogonal fluctuations, respectively.

2.1.1. Fourier Transformation and Cumulant Generating Function

For an arbitrary function $f_i(x)$, the Fourier transformations are written as

$$\tilde{f}_i(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}x e^{-ipx} f_i(x), \tag{7}$$

$$f_i(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}p e^{ipx} \tilde{f}_i(p). \tag{8}$$

Fourier image $\tilde{f}_i(p)$ of the probability distribution function $f_i(x)$ can be calculated explicitly in the terms of cumulants as

$$\tilde{f}_i(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}x e^{-ipx} f_i(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\mathcal{H}\left(p; \left\{\langle x^n \rangle_C\right\}_{n=1}^{\infty}\right)\right\}, \tag{9}$$

where the function $\mathcal{H}\left(p;\left\{\left\langle x^n\right\rangle_{\mathrm{C}}\right\}_{n=1}^{\infty}\right)$ is cumulant generating function, which is sometimes also called the second characteristic function and plays a similar role as the Hamiltonian in quantum statistical mechanics, and $\left\langle \boldsymbol{x}^{\boldsymbol{n}}\right\rangle_{\mathbf{C}}$ denotes $\boldsymbol{n}$-th order cumulant of a random variable $\boldsymbol{x}$ concerning the probability distribution function $f_i(\boldsymbol{x})$

$$\left\langle \boldsymbol{x}^{\boldsymbol{n}}\right\rangle_{\mathbf{C}} = \frac{\mathbf{d}^{\boldsymbol{n}}}{\mathbf{d}^{\boldsymbol{n}}\boldsymbol{\lambda}} \ln \int_{\mathbf{C}_x} \mathbf{d}x \mathbf{e}^{\lambda x} f_i(\boldsymbol{x})\Bigg|_{\lambda=0}. \tag{10}$$

Inserting eq. (8) into eq. (5) to obtain explicit expression of the resulting probability distribution function is a simple and straightforward task as

$$\begin{aligned} f_f(x) &= \mathcal{N}^{-1}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{d}p\tilde{f}_i(p)\int_{\mathrm{C}_\varepsilon}\mathrm{d}\varepsilon\int_{\mathrm{C}_\zeta}\mathrm{d}\zeta\, g_{\mathrm{orth}}(\varepsilon)g_{\mathrm{in}}(\zeta)\mathrm{e}^{ip\sqrt{(x-\zeta)^2+\varepsilon^2}} \\ &= \mathcal{N}^{-1}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{d}p\tilde{f}_i(p)\mathcal{F}_n(x,p), \end{aligned} \tag{11}$$

where immediate function $\mathcal{F}_n(x,p)$ denotes the integration overall degrees of freedom of fluctuations

$$\mathcal{F}_n(x,p) = \int_{\mathrm{C}_\varepsilon}\mathrm{d}\varepsilon\int_{\mathrm{C}_\zeta}\mathrm{d}\zeta\, g_{\mathrm{orth}}(\varepsilon)g_{\mathrm{in}}(\zeta)\mathrm{e}^{ip\sqrt{(x-\zeta)^2+\varepsilon^2}}, \tag{12}$$

and normalized constant $\mathcal{N}$ is

$$\mathcal{N} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathrm{d}x\int_{-\infty}^{\infty}\mathrm{d}p\tilde{f}_i(p)\mathcal{F}_n(x,p). \tag{13}$$

In general, it is hard to find a universal analytical solution for the immediate function $\mathcal{F}_n(x,p)$ even with very simple but meaningful probability distribution functions of both fluctuations, ζ and ε, such as the Gaussian distribution function.

3. Analytical Properties of Final Probability Distribution Function $f_f(x)$

To investigate the analytical properties of the final probability distribution function $f_f(\boldsymbol{x})$, it would be useful to recall it in the most abstract form as eq. (5). Its first-order derivative is

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}f_f(x) &= \mathcal{N}^{-1}\int_{C_\varepsilon}\mathrm{d}\varepsilon\int_{C_\zeta}\mathrm{d}\zeta\, g_{\mathrm{orth}}(\varepsilon)\, g_{\mathrm{in}}(\zeta)\frac{\mathrm{d}}{\mathrm{d}x}f_i\left(\sqrt{(x-\zeta)^2+\varepsilon^2}\right)\\ &= \mathcal{N}^{-1}\int_{C_\varepsilon}\mathrm{d}\varepsilon\int_{C_\zeta}\mathrm{d}\zeta\, g_{\mathrm{orth}}(\varepsilon)\, g_{\mathrm{in}}(\zeta)\frac{\mathrm{d}}{\mathrm{d}z}f_i(z)\bigg|_{z=\sqrt{(x-\zeta)^2+\varepsilon^2}}\frac{x-\zeta}{\sqrt{(x-\zeta)^2+\varepsilon^2}}.\end{aligned} \tag{14}$$

It is not easy to draw the whole picture of the analytical properties of the final probability distribution function $f_f(x)$ from its first derivative. However, by considering separated contributions of each kind of fluctuation, it can show the effects of fluctuations on the final probability distribution function $f_f(x)$. The detailed analysis of total contributions of both fluctuations is out of the scope of present work and will be discussed in another work where the full framework would be studied. As the purpose of the present work, the contribution of orthogonal fluctuations constituting the symmetry breaking is analyzed below.

3.1. *The orthogonal fluctuation*

In the case where the in-space fluctuation is off, the eq. (14) should read

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}x}f_f(x) &= \mathcal{N}^{-1}\int_{C_\varepsilon}\mathrm{d}\varepsilon\, g_{\mathrm{orth}}(\varepsilon)\frac{\mathrm{d}}{\mathrm{d}x}f_i\left(\sqrt{x^2+\varepsilon^2}\right)\\ &= x\left(\mathcal{N}^{-1}\int_{C_\varepsilon}\mathrm{d}\varepsilon\, g_{\mathrm{orth}}(\varepsilon)\frac{\mathrm{d}}{\mathrm{d}z}f_i(z)\bigg|_{z=\sqrt{x^2+\varepsilon^2}}\frac{1}{\sqrt{x^2+\varepsilon^2}}\right)\end{aligned} \tag{15}$$

The second line of eq. (15) implies that the first derivative of the final probability distribution function $f_f(x)$ tends to zero when x goes to zero, i.e. at $x=0$, the function $f_f(x)$ gets a local extremum. Therefore, the direct consequence of this result is that an orthogonal fluctuation will cause any symmetric probability distribution function at $x=0$, i.e. $f_i(x)=f_i(|x|)$ to transform to a new one having local extremum at $x=0$, i.e. $f_f(x)=f_f(|x|)$ and

$$\frac{\mathrm{d}}{\mathrm{d}x}f_f(x)\bigg|_{x=0}=0. \tag{16}$$

This primary conclusion confirms that the transition from Boltzmann distribution function $f_{\mathrm{B}}(x)=\frac{1}{2\lambda}\mathrm{e}^{-\frac{|x|}{\lambda}}$ to Gaussian distribution function $f_{\mathrm{G}}(x)=\frac{1}{\sqrt{2\pi\sigma}}\mathrm{e}^{-\frac{x^2}{2\sigma^2}}$ is undertaken by the mechanism of an orthogonal fluctuation.

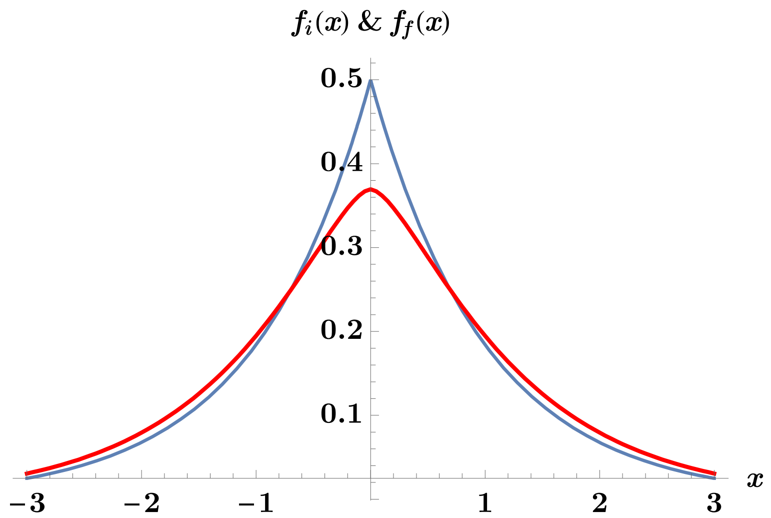


Figure 1. *Graphical representation of the transition from initial symmetric Boltzmann distribution function* $f_{\mathrm{B}}(x)=\frac{1}{2\lambda}\mathrm{e}^{-\frac{|x|}{\lambda}}$ *to Gaussian distribution function* $f_{\mathrm{G}}(x)=\frac{1}{\sqrt{2\pi\sigma}}\mathrm{e}^{-\frac{x^2}{2\sigma^2}}$ *in which under an orthogonal fluctuation the sharp peak of the initial distribution, i.e.* $\frac{\mathrm{d}}{\mathrm{d}x}f_i(0+)\neq\frac{\mathrm{d}}{\mathrm{d}x}f_i(0-)=0$ *transforms to round peak of final distribution* $\frac{\mathrm{d}}{\mathrm{d}x}f_i(0)=0$

4. The transition of Envelop Function of Cooper Pair Wavefunction in Superconducting-Normal Phase Transition

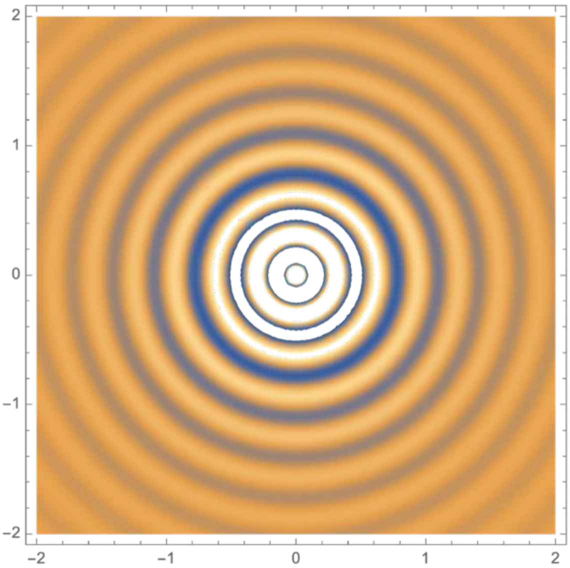
In the long history of the BCS theory, the Cooper pair and its wavefunction as

$$|\Psi\rangle_{\mathrm{C}}=\sum_{k>k_{\mathrm{F}}}\frac{1}{2\varepsilon_{\mathbf{k}}-E}c^{\dagger}_{\mathbf{k}\uparrow}c^{\dagger}_{-\mathbf{k}\downarrow}|F\rangle, \tag{17}$$

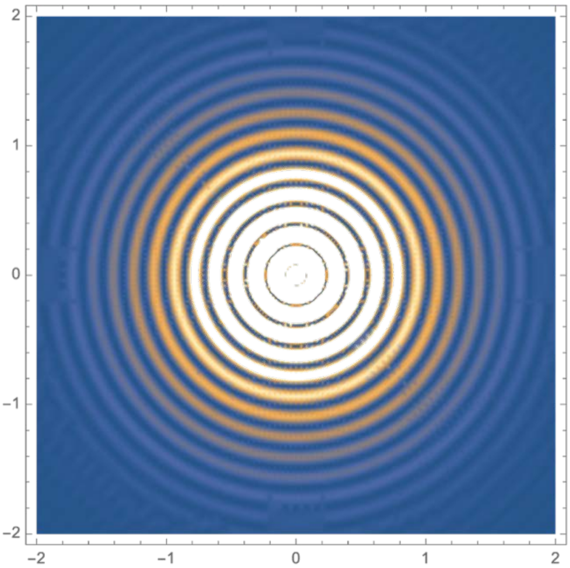
have been usually analyzed in the momentum-space. The role of the Fermi sea, $|F\rangle=\prod_{k<k_{\mathrm{F}}}c^{\dagger}_{\mathbf{k}\uparrow}c^{\dagger}_{-\mathbf{k}\downarrow}|0\rangle$, is to Pauli-block states below the Fermi energy ε_{F} (Ortiz and Dukelsky, 2006; Waldram, 1996). The first investigation of Cooper pair in coordinate-space

$$\begin{aligned}\Psi_{\mathrm{C}}(r) &\propto\sum_{k}u_kv_k\exp(ikr)\\ \Psi_{\mathrm{C}}(r) &\propto\sum_{k}\frac{\cos(kr)}{\sqrt{\varepsilon^2+\Delta^2}}\\ \Psi_{\mathrm{C}}(r) &=N(0)\int\frac{\cos(kr)d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2}},\end{aligned} \tag{18}$$

was implemented in the work (Kadin, 2007), where it was shown that this leads to an internal spherically symmetrical quasi-atomic wavefunction, with an identical "onion-like" layered structure for each of the electrons constituting the Cooper pair. In the expression eq. (18), the sum is taking over all $\mathbf{k}$--states near $\mathbf{k}_F$, $\varepsilon_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$, where $\epsilon_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m}$ and $\mu = \frac{\hbar^2 \mathbf{k}_F^2}{2m}$ are the free electron kinetic energy and the Fermi energy, and Δ is the BCS superconducting energy gap. The functions $u_{\mathbf{k}}$ and $v_{\mathbf{k}}$ are the standard variational parameters of the BCS theory, such that $2u_{\mathbf{k}}v_{\mathbf{k}} = \left(\varepsilon_{\mathbf{k}}^2 + \Delta^2\right)^{-\frac{1}{2}}$, and the $N(0)$ is the density of states for a single electron spin per unit energy at the Fermi level.



(a) Graphical representation of an internal spherically symmetrical quasi-atomic wavefunction of Cooper pair $\psi_C(x)$ in coordinate-space



(b) Graphical representation of an internal spherically symmetrical quasi-atomic wavefunction of Cooper pair density $|\psi_C(x)|^2$ in coordinate-space

Figure 2. *Graphical representation of Cooper pair and its density in coordinate-space*

4.1. Cooper Pair Wavefunction in Coordinate Presentation

Follow the standard routine of calculus manipulation (Kadin, 2007), the internal structure of Cooper pair wavefunction which is also called the singlet pairing function or the Gorkov's wavefunction is obtained and given by

$$\Psi_C(r) = \mathcal{N}_C \cos(k_F r) K_0\left(\frac{r}{\pi\xi_0}\right), \tag{19}$$

where $\mathcal{N}_C$ is the normalized constant, $K(z)$ is the complete elliptic integral of the first kind, k_F is the Fermi wavevector at the surface of the Fermi sea and $K_0(z)$ is the zero-order modified Bessel function with an asymptotic form that is similar to an exponential $K_0(x) \sim (\pi / 2x)^{1/2} \exp(-x)$ for large x 0. The function $K_0(z)$ has a weak divergence $z = 0$, which must be cut off by choosing a cutoff energy scale. In the BCS theory, this cutoff is usually given by an energy comparable to the Debye energy $\hbar\omega_D$, which is much larger than the energy gap Δ.

Denoting dimensionless variable x

$$x = \frac{r}{\pi\xi_0}, \tag{20}$$

and parameter γ

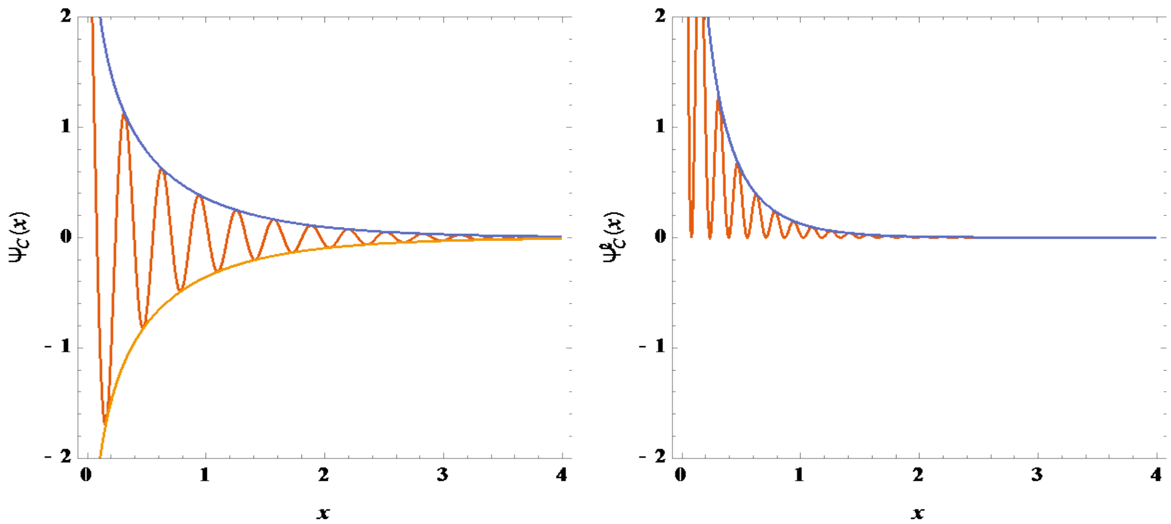
$$\gamma = \pi k_F \xi_0, \tag{21}$$

the Cooper pair wavefunction is rewritten as

$$\Psi_C(x) = \mathcal{N}_C \cos(\gamma x) K_0(x), \tag{22}$$

in which normalized constant $\mathcal{N}_C$ is now exactly obtained in term of γ as

$$\mathcal{N}_C = 2\sqrt{\frac{2}{\pi\left(2K\left(-\gamma^2\right)+\pi\right)}}. \tag{23}$$



(a) Graphical representation of Cooper pair wavefunction $\Psi_C(x)$ in coordinate-space with $\gamma = 20$

(b) Graphical representation of Cooper pair density $|\Psi_C(x)|^2$ in coordinate-space.

Figure 3. *Graphical representation of Cooper pair and its density in coordinate-space with $\gamma = 20$*

The Cooper pair wavefunction in coordinate representation corresponds to the standing wave, with a spatial modulation γ. Typically, the values of the superconducting system are approximately

$$\begin{aligned} k_{\mathrm{F}}^{-1} &\sim 0.1\,\mathrm{nm}, \\ \xi_0 &\sim 100\,\mathrm{nm}, \end{aligned} \tag{24}$$

and these waves rapidly oscillate around k_{F}, modulated by slowly varying envelope function with a characteristic scale off ξ_0. For graphical representation which is shown in fig. 3, it would take an empirical value

$$k_{\mathrm{F}}\xi_0 = \frac{20}{\pi}, \tag{25}$$

or

$$\gamma = 20, \tag{26}$$

and corresponding normalized constant

$$\begin{aligned} \mathcal{N}_{\mathrm{C}} &= 2\sqrt{\frac{2}{\pi\left(2K\left(-\gamma^2\right)+\pi\right)}}\Bigg|_{\gamma=20} \\ &= 0.843439. \end{aligned} \tag{27}$$

It is not difficult to realize that the envelop function of Cooper pair wavefunction $\sim K_0(x)$ is very similar to Boltzmann function for large z. In superconducting state, both electrons of the Cooper pair would be expected to have the same spatial wavefunction, and hence the same quasi-static charge distribution. This corresponds to a spherical layered charge distribution of the Cooper pair, with periodic layers spaced by $\sim 2\,\mathring{\mathrm{A}}$(Kadin, 2007), as shown in the fig. 2.

4.2. The Contribution of The Orthogonal Fluctuations to Cooper Pair Wavefunction in Coordinate Presentation

In superconductivity, the thermal fluctuations $\sim\left(T-T_C\right)$ are the most considerable ones causing the superconductivity - normal phase transition when the system temperature is nearby critical T_C. The contribution of thermal fluctuation in Cooper pair wavefunction is naturally of the orthogonal fluctuations since they appear in the energy equation in square form as

$$\varepsilon^2 + \Delta^2 \to \varepsilon^2 + \Delta^2 + \epsilon^2. \tag{28}$$

Using the immediate expression eq. (16), it would be seen that the weak singularity of the envelope function $\sim K_0(z)$ at $z=0$ z = 0 (Kadin, 2007) of Cooper pair wavefunction should be canceled to take finite value reflecting the finiteness of quasi-static charge distribution. It is a great work of an orthogonal fluctuation without a cutoff of energy scale as it is usually done in BCS theory.

To study the contribution of the orthogonal fluctuation ϵ to Cooper pair wavefunction in detail, the characteristic parameters of BCS superconducting state in the last subsection are taken into account and the whole calculating procedure in (Kadin, 2007) is carefully repeated. In this stage of consideration, due to the non-analyticity of the integrals, some numerical calculations are performed.

In the regime of small fluctuation ϵ where its variance σ_ϵ is assumed much smaller than the BCS energy gap Λ, the envelop function of Cooper pair wavefunction transforms from weak singularity Boltzmann-like function to round peak Boltzmann-like function in large values of distance. By increasing σ_ϵ to approach to its physical maximum, BCS energy gap Λ, the envelope function under consideration becomes more Gaussian-like function.

The integration over all the possibles σ_ϵ in the range from zero to BCS energy gap Λ delivers a final picture of enveloping function of Cooper pair under influence of thermal orthogonal fluctuations in the superconducting state nearby critical temperature T_C in fig. 4.

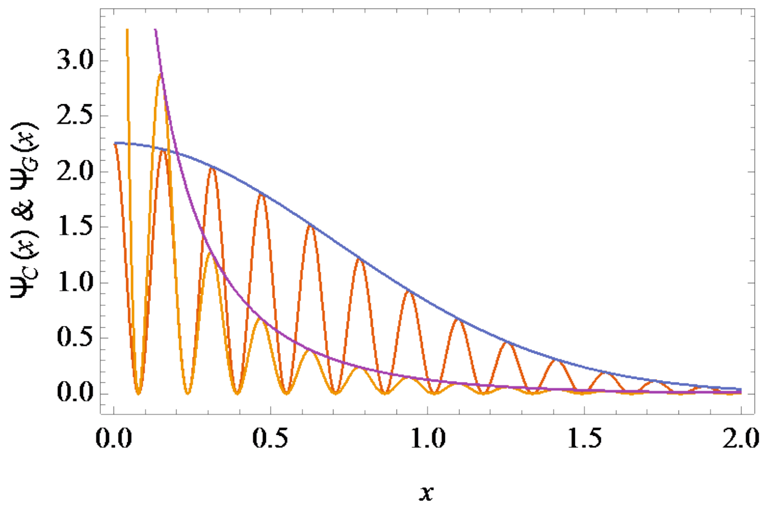


Figure 4. *Graphical representation of Cooper pair wavefunction and its envelop function $\psi_C(x)$ (sharp peak in x=0) and $\psi_G(x)$ (round peak when x=0) without and with the contributions of thermal orthogonal fluctuations, respectively*

In the last picture fig. 4, the round peak enveloped wavefunction is very similar to Gaussian wave packet

$$\Psi_G(z) \sim \cos(\gamma' x)\exp\left(-\frac{1}{2}x^2\right), \tag{29}$$

describing a standing wave of electrons which requires the spherical invariance of the space. The parameter γ' would be chosen to be fitted to characteristic parameters of a given superconducting state, in the case, $\gamma' = \gamma$.

Fig. 4 provides that the contribution of thermal orthogonal fluctuations around critical temperature T_C forces the envelop function of Cooper pair to transform from Boltzmann-like function to Gaussian ones, i.e. the symmetry of the system changes from the translation-like invariance to the spherical invariance. This means the happening of superconductivity - normal phase transition.

5. Conclusion

In this work, we have attempted to finger out why and how the envelope function of Cooper pairs of superconductivity would change to the Gaussian wave packet utilizing orthogonal thermal fluctuations.

The analytical and numerical results done in the work show that the contribution of an orthogonal fluctuation eliminates the mathematical artifacts of singularity in quasi-static charge distribution, which are not enabled in the BCS theory without the cutoff of energy scale, also the integration of all possible contributions of thermal orthogonal fluctuations supplies a possible transition of Cooper pair envelop function in the phase transition from superconducting phase to normal conducting phase.

We suppose a possible connection between geometrical invariant of space and envelope functions. The Boltzmann-like form of envelope function of Cooper pair corresponds to the free moving with plan wavefronts, and the symmetry property of space is translation invariant. The Gaussian form of envelope function of the Cooper pair corresponds to the bound state with spherical wavefronts near some fixed point, and the symmetry property of space is spherical invariant.

The transformation of envelope functions might correspond to the changing of symmetrical behavior in the space of a Cooper pair from its normal to superconducting states near the phase transition critical temperature T_C. In the superconducting state, the Cooper pair can move as free quasiparticles without resistance, i.e. translation symmetry of the space has occurred. The normal state, where electrons can not move freely, will be corresponded the spherical symmetry of the space.

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DỊCH CHUYỂN PHÂN BỐ DƯỚI TÁC ĐỘNG CỦA CÁC THĂNG GIÁNG NGẪU NHIÊN VUÔNG GÓC: ỨNG DỤNG CHO CHUYỂN PHA SIÊU DẪN - KIM LOẠI

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TÓM TẮT

Một số hàm phân bố mật độ xác suất (probability density function – PDF) của biến ngẫu nhiên liên hệ chặt chẽ với tính đối xứng của biến ngẫu nhiên. Các hàm phân bố mật độ xác suất Boltzmann và Gaussian bất biến dưới phép biến đổi tịnh tiến và cầu tương ứng của biến số, là những ví dụ đã được nghiên cứu đầy đủ, phản ánh không chỉ tính đối xứng của nhiều hiện tượng vật lí mà cả các định luật bảo toàn tiềm ẩn. Trong vật lí thống kê và nhiều lĩnh vực của hệ phức hợp, sự biến đổi từ phân bố mật độ xác suất từ dạng Boltzmann sang dạng Gaussian xuất hiện khá phổ biến. Những hiện tượng quan sát được này cung cấp bằng chứng về sự chuyển pha, cụ thể là chuyển pha loại hai, xuất hiện khi tính đối xứng của một đại lượng vật lí trong hệ bị phá vỡ. Mục đích của bài báo này là nghiên cứu loại dịch chuyển trên trong siêu dẫn thông qua khảo sát sự chuyển từ hàm bao của hàm sóng điện tử và cặp Cooper trong không gian tọa độ tương ứng với sự biến đổi hành vy đối xứng của không gian từ trạng thái dẫn sang trạng thái siêu dẫn tại vùng gần nhiệt độ chuyển pha

Keywords: siêu dẫn; chuyển pha; thăng giáng vuông góc