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Research Article

INITIAL-STATE PHOTON RADIATION TO $e^{-}e^{+} \rightarrow W^{-}W^{+} \rightarrow 4$ JETS AT LEPTON COLLIDERS

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ABSTRACT

In this article, we calculate QED corrections due to the initial-state photon radiation (ISR) to process $e^{-}e^{+} \rightarrow W^{-}W^{+} \rightarrow 4$ jets at lepton colliders. The computations are performed up to the correction with order $\mathcal{O}(\alpha^{3})$. For physically interpretation, the impact of ISR corrections on total cross sections and its relevant distributions are studied. We find that the corrections are almost 10% of the total cross-section. Thus, these corrections are significant, and they must be taken into account at lepton colliders in the future.

Keywords: Electron-positron colliders; QED corrections; W-pair productions; Initial State Radiation; Numerical computations

1. Introduction

The process $e^{-}e^{+} \rightarrow W^{+}W^{-} \rightarrow 4$ jets plays a crucial role at lepton colliders. Since the reactions provide the most precise direct determination of the mass of W- boson (M_W), one of the most important parameters in the Standard Model (SM). The precise measurement for M_W plays a key role in updating the global SM fit. From kinematic fit, we can test the SM at high energies and extract the new physics contributions. Furthermore, we can search for the coupling of triplet gauge bosons from the corrected cross-section of the process. As a result the structure of non-Abelian gauge theories (Baer el at., 2013) is then verified. Last but not least, we can probe the coupling of the Higgs boson to W-pair through the production cross-sections.

Higher-order quantum corrections to W-pair production are necessary for matching future high precision data at lepton colliders. There have been available many computations for one-loop electroweak radiative corrections to the reaction at lepton colliders (Bohm el at., 1988; Fleischer, Jegerlehner & Zralek, 1989; Beenakker & Denner, 1994; Denner, Dittmaier, Roth, & Wackeroth, 2000a; 2000b; K"uhn, Metzler, & Penin,

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2008). Full one-loop electroweak radiative corrections to the W-pair production including the initial beam polarization effects have performed in Ref. Phan, Nguyen and Nguyen (in press). The evaluations for full one-loop electroweak radiative corrections to four fermion productions have found (Denner, Dittmaier, Roth, & Wieders, 2005a; 2005b). However, there have not been considered higher-order QED corrections in the above papers. In present work, we calculate the QED corrections due to the initial state radiation up to order $\mathcal{O}(\alpha^3)$ to the process $e^-e^+ \to W^+W^- \to 4$ jets at lepton colliders. For numerical results, the impact of the ISR corrections on total cross sections and their important distributions are studied.

This report is organized as follows. In Section 2, we present the calculation in detail. The structure-function methods which are used for modeling QED corrections to this reaction are discussed. The physical predictions for $e^-e^+ \to W^+W^- \to 4$ jets are generated in Section 3. Conclusions and prospects will be devoted to Section 3.

2. The calculations

Two methods for modeling ISR corrections to the process $e^-e^+ \to W^+W^- \to 4$ jets are considered in this paper. First, we follow the approach in (Fujimoto, Igarashi, Nobuya, Yoshimitsu, & Keijiro, 1990). In this approach, a total cross-section including ISR corrections up to order of $\mathcal{O}(\alpha^2)$ are computed as follows:

$$\sigma_{\rm rad}(s) = \int_0^1 dx\, \mathcal{H}(x, Q^2)\, \hat{\sigma}_0(s(1-x)). \tag{1}$$

Here, $\mathcal{H}(x, Q^2)$ is a radiator at energy scale Q^2 which is defined by formula (11.213) (Fujimoto el at., 1990). It is given:

$$\begin{aligned}\mathcal{H}(x, Q^2) = \Delta\beta x^{\beta-1} - \beta(1-\frac{x}{2}) \\ + \frac{\beta^2}{8}\Big[-4(2-x)\ln x - \frac{1+3(1-x)^2}{x}\ln(1-x) - 6 + x\Big]\end{aligned} \tag{2}$$

with

$$\beta = \frac{2\alpha}{\pi}\Big(\ln(\frac{Q^2}{m_e^2}) - 1\Big), \quad \Delta = 1 + \frac{\alpha}{\pi}\Big(\frac{3}{2}\ln(\frac{Q^2}{m_e^2}) + \frac{\pi^2}{3} - 2\Big),$$

where α is the fine structure constant, m_e is the electron mass. In Eq. (1), $\hat{\sigma}_0(s(1-x))$ is tree-level cross sections for the process $e^-e^+ \to W^+W^- \to 4$ jets at reduced center-of-mass energy $s(1-x)$ with the momentum fraction x.

Based on an alternative method, following the factorization theorems, the total cross-section for the process $e^-e^+ \to W^-W^+ \to 4$ jets including the initial-state QED corrections can be expressed as a convolution of the two structure functions (SF) for two beams and of the lowest-order cross-section:

$$\sigma_{\rm SF}(s) = \int dx_1 dx_2 D(x_1, Q^2) D(x_2, Q^2) \hat{\sigma}_0(x_1 x_2 s). \tag{3}$$

Here, $D(x,Q^2)$ is the non-singlet collinear structure-function for modeling the initial-state photon radiation at the energy scale Q^2. This function explains the probability of finding an electron possessing momentum fraction x at the energy scale Q^2 inside an electron parent.

The structure-function applied in this work is written as follows:

$$D_{GL}(x,Q^2) = \frac{\exp\left[\frac{1}{2}\beta\left(\frac{3}{4}-\gamma_E\right)\right]}{\Gamma\left(1+\frac{1}{2}\beta\right)}\frac{1}{2}\beta\,(1-x)^{\frac{1}{2}\beta-1}\,. \tag{4}$$

Here, Γ is the Gamma function; γ_E is the Euler-Mascheroni constant. In this work, the additive SF with the third finite order is then obtained (Cacciari, Deandrea, Montagna, & Nicrosini, 1992):

$$D(x,Q^2) = \sum_{i=0}^{3} d_A^{(i)}(x,Q^2) \tag{5}$$

with

$$d_A^{(0)}(x,Q^2) = D_{GL}(x,Q^2), \tag{6}$$

$$d_A^{(1)}(x,Q^2) = -\frac{1}{4}\beta(1+x), \tag{7}$$

$$d_A^{(2)}(x,Q^2) = \frac{1}{32}\beta^2\left[(1+x)\left(-4\ln(1-x)+3\ln(x)\right)-4\frac{\ln x}{1-x}-5-x\right], \tag{8}$$

$$\begin{aligned} d_A^{(3)}(x,Q^2) = \frac{1}{384}\beta^3\Bigg\{&(1+x)\left[18\zeta(2)-6\mathrm{Li}_2(x)-12\ln^2(1-x)\right] \\ &+\frac{1}{1-x}\Bigg[-\frac{3}{2}(1+8x+3x^2)\ln x-6(x+5)(1-x)\ln(1-x) \\ &-12(1+x^2)\ln x\ln(1-x)+\frac{1}{2}(1+7x^2)\ln^2 x \\ &-\frac{1}{4}(39-24x-15x^2)\Bigg]\Bigg\}. \end{aligned} \tag{9}$$

In this formula, ζ is the Riemann zeta-function and Li_2 is the dilogarithm function.

In this calculation, the tree-level cross sections are generated by using the GRACE program (Ishikawa el at., 1993). For example, the process $e^-e^+ \to W^-W^+ \to u\bar{u}d\bar{d}$ consists of 159 Feynman diagrams. In Figure (1), we show some typical Feynman diagrams. GRACE program provides the internal numerical checks for the calculation, e.g., a test of gauge invariance. After checking successfully the generated codes, we are going to discuss the numerical results in the next section.

3. Numerical results

We use the following input parameters for the calculation. The Higgs boson mass is $M_H = 126$ GeV. Because of the limited accuracy of the measured value for M_W, we hence take the value that is derived from the electroweak radiative corrections to the muon decay width (Δr) (Hioki, 1996) with $G_\mu = 1.16639 \times 10^{-5}$ GeV^{-2}. As a result, M_W is a function of M_H.

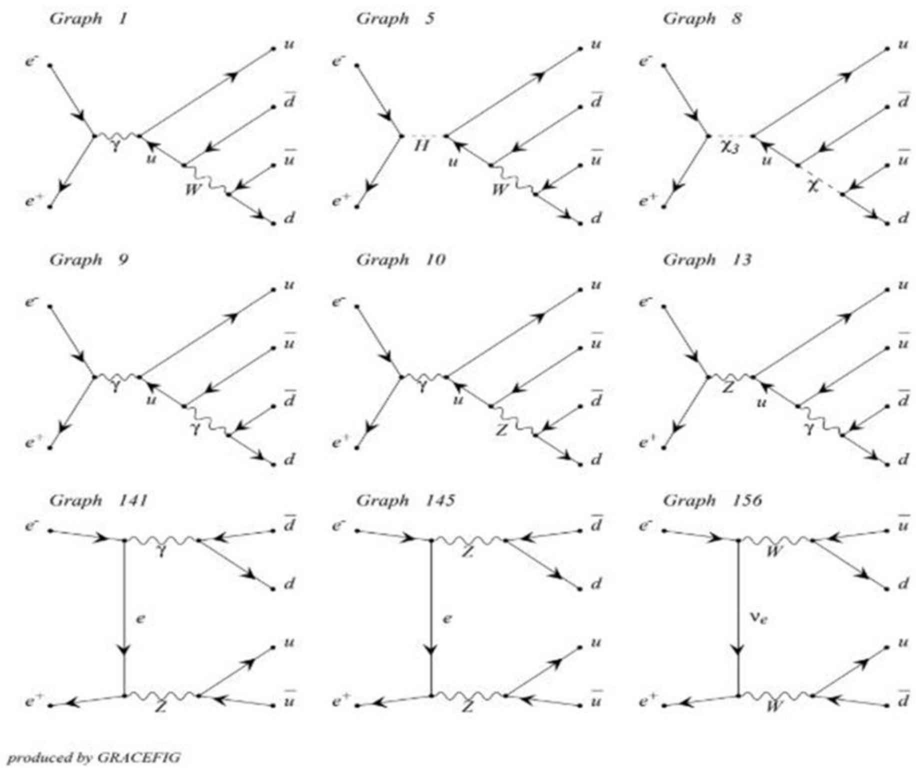

Figure 1. *Typical Feynman diagrams*

The resulting $M_W = 80.370$ GeV is corresponding to $\Delta r = 2.49\%$. The mass of Z boson is 91.1876 GeV. Finally, for the lepton masses, we take $m_e = 0.51099891$ MeV, $m_\mu = 105.658367$ MeV, and $m_\tau = 1776.82$ MeV. For the quark masses, we take $m_u = 63$ MeV, $m_d = 63$ MeV, $m_c = 1.5$ GeV, $m_s = 94$ MeV, $m_t = 173.5$ GeV, and $m_b = 4.7$ GeV. In this work, the QED corrections are evaluated in the so-called G_μ scheme. In details, the electroweak coupling is derived as follows:

$$\alpha = \sqrt{2} G_\mu M_W^2 (1 - M_W^2/M_Z^2)/\pi. \quad (10)$$

Moreover, we also apply the following cuts for the final jets:

$$|\cos\theta_{\rm jets}| \leq 0.985, \quad E_{\rm jets} \geqslant 1 \text{ GeV}, \quad |\cos\theta_{\rm 2jets}| \leq 0.985. \quad (11)$$

To study the impact of the QED corrections, we define the corrections as follows:

$$\delta[\%] = \frac{\sigma_{\rm rad/SF}(s) - \sigma_0(s)}{\sigma_0(s)} \times 100. \quad 12)$$

For all numerical results shown in this paper, the notations **rad** and **SF** present for the QED corrections using the structure functions in Eq. (2) and Eq. (5) respectively. In Figs.2, the total cross-section (left panel) and QED corrections (right panel) are presented as a function of center-of-mass energy $\sqrt{s}$. The $\sqrt{s}$ varies from 200 GeV to 1000 GeV. In these figures, the dashed line with rectangle points presents for tree-level cross-section for this process (without the ISR corrections). The solid line with triangle points is shown for the $\mathcal{O}(\alpha^3)$ corrections which computed from Eq. (3) while the dashed line with circle points presents the $\mathcal{O}(\alpha^2)$ ISR corrections as defined in Eq. (1). Total cross sections decrease similar to the function of s^{-1} in the region $200 \leq \sqrt{s} \leq 1000$ GeV. In the right panel figure, ISR corrections to this process are shown. We observe that the corrections are a range of $\sim -30\%(\sim -20\%)$ to $\sim -7\%(\sim -12\%)$ for **rad** (and **SF**) respectively.

The differential cross-sections are shown as functions of $E_{\rm jet}$, $\cos\theta_{\rm jet}$ at $\sqrt{s} = 340$ GeV. In the left figures, we present differential cross sections concerning $E_{\rm jet}$ and its ISR corrections. The peak is found around $E_{\rm jet} = 20$ GeV which is corresponding to the threshold of W-pair production. The cross-section is then decreased beyond this peak. In the right panel figures, the differential cross sections as a function of $\cos\theta_{\rm jet}$ and its ISR corrections are plotted. We observe the massive contributions (order of 10%) of ISR corrections to these distributions. They must be taken into account at future colliders.

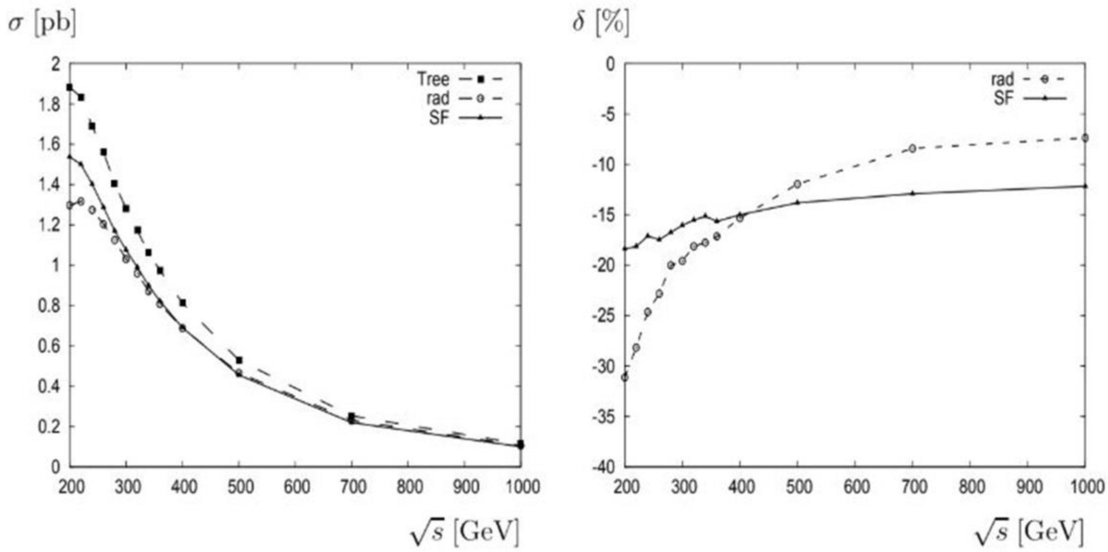

Figure 2. *Total cross sections and ISR corrections are presented as a function of center-of-mass energy*

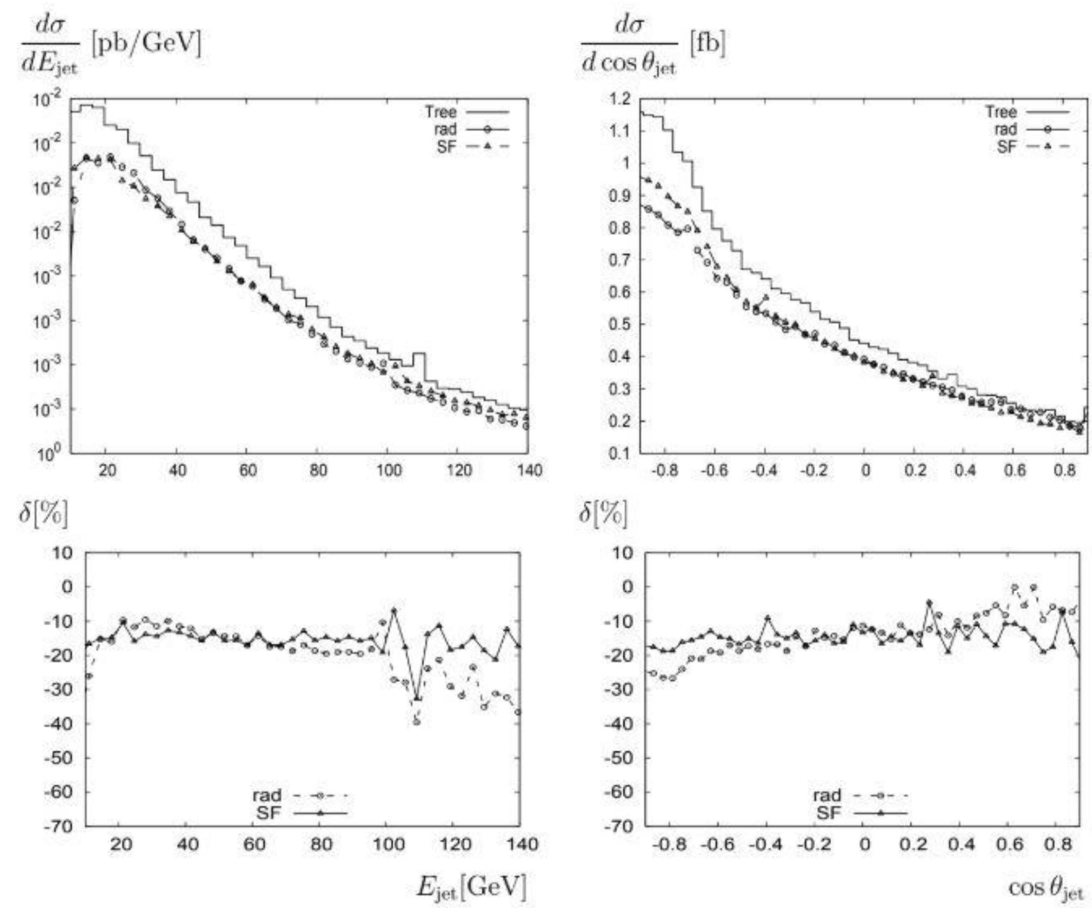

Figure 3. *Differential cross-sections with respect to* $E_{\rm jet}$ *and* $\cos\theta_{\rm jet}$ *at* $\sqrt{s} = 340$ *GeV*

4. Conclusions

We have presented the QED corrections up to $\mathcal{O}(\alpha^3)$ due to the initial-state photon radiation to the processes $e^-e^+ \to W^-W^+ \to 4$ jets at future lepton colliders. In this research, we have investigated the effect of the ISR corrections on the total cross sections and its relevant distributions. We have found that the corrections are an order of 10% contributions. The corrections are massive contributions which must be taken into account for measuring the properties of vector bosons at future lepton colliders.

- ***Conflict of Interest:*** *Authors have no conflict of interest to declare.*

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BỨC XẠ PHOTON TRẠNG THÁI ĐẦU CỦA QUÁ TRÌNH $e^+e^- \to W^+W^- \to 4$ JETS TẠI CÁC MÁY VA CHẠM LEPTON

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TÓM TẮT

Trong bài báo này, chúng tôi tính toán bổ chính QED dựa trên bức xạ photon ở trạng thái ban đầu (ISR) cho quá trình $e^+e^- \to W^+W^- \to 4$ jets tại các máy va chạm lepton. Các tính toán được thực hiện tới bổ chính bậc $\mathcal{O}(\alpha^3)$. Trong các kết quả vật lí, chúng tôi nghiên cứu tác động của bổ chính ISR trên tiết diện tán xạ toàn phần và các phân bố tiết diện tán xạ. Các bổ chính cho đóng góp khoảng 10% là rất đáng kể. Do đó, chúng phải được tính toán đến trong phân tích thực nghiệm tại các máy va chạm lepton trong tương lai.

Từ khóa: máy va chạm electron-positron; hiệu chỉnh QED; bức xạ trạng thái ban đầu; tính toán số