



Research Article

THE DERIVATIVE FUNCTION IN MATHEMATICS EDUCATION IN HIGH SCHOOL

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ABSTRACT

Calculating the derivative of a function f gives an expression, it also is a function. The derivative function f' and the function f are interrelated from an aspect of graphs and algebraic expressions. The article shows the incomplete relationship between the initial function and the derivative function present in the current curriculum and textbooks of Mathematics High School in Vietnam. This suggests new types of tasks that can appear in analytics teaching and are suitable for multiple-choice questions.

Keywords: derivative function; graph; calculus teaching

1. Define the derivative function

The derivative function f' have mentioned here is the first derivative.

According to Calculus Early Transcendentals Seventh Edition of Stewart (2012), the derivative definition formula $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ * allows you to look at the derivative as a function with the variable x . The function f' is defined as follows:

Given any number x for which this limit exists, we assign to x them number $f'(x)$. So we can regard f' as a new function, called the derivative of f and defined by Equation (*). We know that the value of f' at x ; $f'(x)$, can be interpreted geometrically as the slope of the tangent line to the graph of f the point $(x; f(x))$.

The function f' is called the derivative of f because it has been “derived” from f by the limiting operation in Equation (*). The domain of f' is the set $\{x | f'(x) \text{ exists}\}$ and may be smaller than the domain of f .

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There are relationships between f and f' , both directions from f to f' and from f' to f as seen from the aspect of graphs and algebraic expressions. The geometric meaning of the derivative - the tangent angle coefficient - is mobilized to build most of these relationships.

1.1. The relationship between f and f' in the direction from f to f'

R1 $f \rightarrow f'$: Given the algebraic expression of f , use the formula defines the derivative by the limit, we can define the derivative expression. From this perspective, it can be seen that: the domain of f' is a subset of the domain of f .

R2 $f \rightarrow f'$: Given the graph of f , we can sketch the graph of f' from the geometric meaning of the derivative (the derivative is the slope of the tangent). Especially:

- At the points where the tangent line of f is horizontal, the graph of the derivative cuts the horizontal axis at those points;
- At the points where the slope of the tangent of the graph f is positive (negative) the value of the positive (negative) derivative.

R3 $f \rightarrow f'$: The function f is not continuous at any point, the derivative function f' is not defined at that point.

R4 $f \rightarrow f'$: At points where f is continuous, f' can be determined whether or not. In terms of graph, f' will not be determined at "corner" points or points with vertical tangents line.

1.2. The relationship between f and f' in the direction from f' to f

R1 $f' \rightarrow f$: The function f' determines at point which f is continuous at that point.

R2 $f' \rightarrow f$: If the derivative value is positive (correspondingly negative) on an open interval $(a;b)$, the function f increases (correspondingly decreases) on an open interval.

R3 $f' \rightarrow f$: Suppose the derivative is zero or unknown at c . If f' changes from positive to negative at c then f has local maximum at c . If f' changes from negative to positive at c then f has a local minimum at c . If f' does not change the sign at c , then f hasn't extreme at c .

In a nutshell, the American curriculum has researched f' in relation to the function f viewed from the aspect of graphs and algebraic expressions. Derivative allows you to see the continuity, variability and extreme of the function.

Considering derivative as a function brings a new function, its properties can be built from the function "born" by mobilizing the geometric meaning of the derivative. A review of applications of derivatives based on derivative signs shows its effect on the properties of the function f .

2. Derivative function in Vietnamese textbooks

In Pham Thi Thanh (2016), the author concludes:

The current textbooks (Doan Quynh et al., 2014; Tran Van Hao et al. 2014) desire to mention the relationships between the two functions f and f' , these relationships are mainly viewed from algebraic expressions. Textbooks have not mentioned much about these relationships from the graphical perspective.

The textbooks rarely have the task to mobilize the geometric meaning of the derivative.

2.1. *The relationship in the direction from $f \rightarrow f'$*

Vietnamese textbook refers to relationship R1 $f \rightarrow f'$; R3 $f \rightarrow f'$ and R4 $f \rightarrow f'$. Compared to the US curriculum:

In relation to R1 $f \rightarrow f'$: The textbook does not specify that the domain of f' is a subset of the domain of f .

In relation to R4 $f \rightarrow f'$: In terms of graphical representation, textbooks only refer to the derivative function will not be determined at the "corner" points, not to the points with the vertical tangents.

2.2. *The relationship in the direction from $f' \rightarrow f$*

The textbook refers to all three relationships R1 $f' \rightarrow f$; R2 $f' \rightarrow f$ and R3 $f' \rightarrow f$ with functions given by algebraic expressions without graph representations.

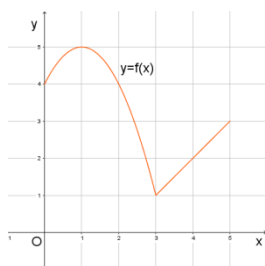
The textbook mentions the relationships between f and f' , these relationships are mainly viewed from algebraic expressions. Textbooks have not mentioned much about these relationships from the graphical perspective.

The method of the textbook is most interested in calculating the derivative of a function given by algebraic expressions by the derivative rule. Textbook due to the lack of attention to the relationships between the two functions seen from the graphical perspective, textbooks rarely have mobilization exercises to derive the geometric meaning of the derivative.

3. Conduct an experiment

From the analysis of Vietnamese textbooks, we propose a hypothesis: "When solving graph-related task types, students have difficulty mobilizing the geometric meaning of derivatives and relationships between the sign of the derivative and the variation of the function". Since then, we have built an experiment with 4 multiple-choice questions. Experimental participation has 28 students of Nguyen Trai High School - Binh Duong.

3.1. *Analysis of question 1*



Question 1: Let the function $y = f(x)$ defined on the interval $(0; 5)$ with the following graphical representation:

1.1. At which point can you approximate the derivative of the function? (Put a cross in the case you choose and answer next)

At points:

Derivative specificity at least 2 points have been shown:.....

It is not possible to approximate derivative at any point because:

1.2. Point out the points at which you know for sure:.....

- The derivative has a positive value:

Explain why you know:

- The derivative is 0:

Explain why you know:

- Negative derivative:.....

Explain why you know:

- No derivative:

Explain why you know:

- I cannot answer the above questions because:.....

The goal of question 1 is to apply the geometric meaning of the derivative - the relationship between the sign of the tangent of the graph f and the value of f' , thus the graph representation is chosen. A graph are performed on the grid to facilitate the drawing and approximate tangent angles. A graph with a part of Parabola and a part of a line makes it difficult to find algebraic expressions for graphs to facilitate the approximate derivative equations with geometric meanings. However, this choice does not completely prevent students from trying to find a two-part algebraic expression corresponding to a graphical representation.

All 28 students could not answer question 1. In which, $5/28$ students said that they could not be calculated because there was no specific function.

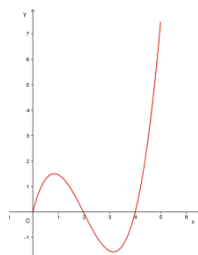
Thus, the student did not mobilize the geometric meaning of the derivative to answer these questions. In particular, students did not realize that the derivative at the extreme point was zero. Besides, we were surprised because no student could use the relationship between the variance of the function and the sign derivative. This is explained by the textbook only interested in the derivative of the function given by algebraic expressions.

3.2. Analysis of question 2

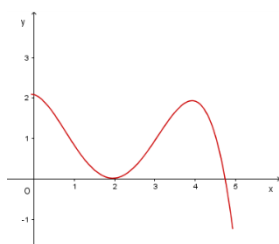
Question 2: Let the function $y = f(x)$ have a derivative function

The derivative function is defined over the interval $(0; 5)$ with the following graph representation:

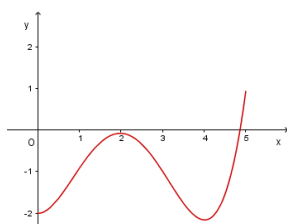
One of the three curves below corresponds to the graph of the function $y = f(x)$.



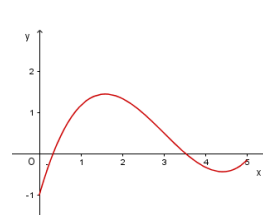
Curves 1



Curves 2



Curves 3



In your opinion, which is the curve?

Reply.

Curve: Because:

The goal of question 2 is to manipulate the relationship between the sign of the derivative and the variation of the function. The functions are represented graphically - graphing to facilitate graph reading strategies. For that, function f' is reading the sign of the value f' and for f student needs to read the intervals of variation. The graph of f' cutting the horizontal axis at 2 points has an integer diaphragm, to facilitate the determination of the intervals of variation. Two curves 1 and 2 have opposite directions, including one correct answer, giving attention to students to compare and test. Curve 3 has variable intervals with non-intact endpoints for students to easily refute. Ask for an explanation of the choice to see the knowledge behind the choice, as there is a chance that the answer will be made by chance. Each option comes with an explanation that shows us the knowledge students apply.

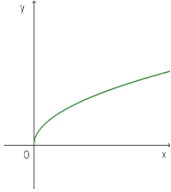
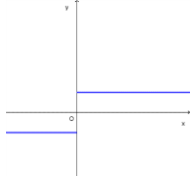
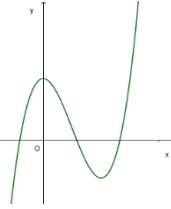

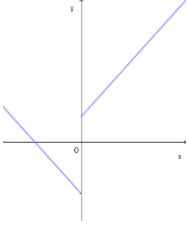
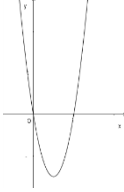

The results of the experiment showed that despite deliberately selecting curve 3 with non-integer variable endpoints, but this curve is the student's most choice. From there, we

have a basis for evaluating half of the students answering by random selection. Students explain based on the domain of functions, even though three curves are an open interval (0; 5). By explaining, we find that the student takes into account the domain of graphs f and f' , however, the student thought that the function f and f' had the same domain. Another student explained based on the similarity of the shape of f and the curve 2. This proves the existence of a misconception among students: f and f' are "uniform".

Thus, none of the 28 students mobilized the relationship between the sign of the derivative and the variation of the function to select the answer.

3.3. Analysis of question 3

Question 3: The first column is a graph of the function $y = f(x)$. The second column is a graph of the randomly presented functions. You show the graph of the derivative corresponding to the graph of the function given.

Graph of the function f	Graph of the function f'
<p>1</p> 	<p>A</p> 
<p>2</p> 	<p>B</p> 
<p>3</p> 	<p>C</p> 
	<p>D</p> 

Please answer by filling the corresponding letters in the box.

- Function 1 - Explain the reason:.....

- Function 2 - Explain the reason:.....

- Function 3 - Explain the reason:.....

The objective of question 3 is that the relationship from the perspective of a graph of two functions with the function f is given properties such as: horizontal tangent, tangential slope increasingly steep as it approaches the point x_0 , the function discontinuous at the point, the interval at which the tangent is positive or negative, the tangent constant.

The graph in this questionnaire is represented on a non-unit coordinate system to prevent the derivation of derivative values. If the approximation of the tangent angle is calculated and compared to the point on the function graph f' , the calculation takes a long time and does not cover the entire set. The problem will be solved if the relationship between f' and f can be applied. Besides, the quantity of graphs in the two columns is not equal, the set of functional graphs f' is more numerous than the set of graphs of f , which forces the student to find the relation of all pairs of graphs. This choice along with the graph of f first and the number of function f less than the function facilitates the strategies for finding the relationship in the direction $f \rightarrow f'$. But strategies recover function f that still have a chance to appear.

Through the results of questionnaire 3, we evaluate that the choice of answers in functions 1 and 2 is random because answers B, C, and D have an approximate number of options. As for function 3 alone, we suspect that there may be students who recognize the formula of function but they do not know how to express their knowledge. Students relate the graph $f \rightarrow$ the expression of $f \rightarrow$ the expression of the derivative function \rightarrow the graph of the derivative function to explain. For example, students identify graph 2 as the graph of the third of degree polynomial.

Students predict the function's expression. This is explained that they have studied functional surveys. Also, students knew the function 3 so they can predict.

This phenomenon is consistent with the institutional relationship with the derivative function in teaching mathematics in Vietnam. This information also helps researchers to develop teaching projects to supplement the missing areas of derivative function knowledge. In particular, this strategy can help students refute the wrong view that has occurred: the initial function and the "homogeneous" derivative function.

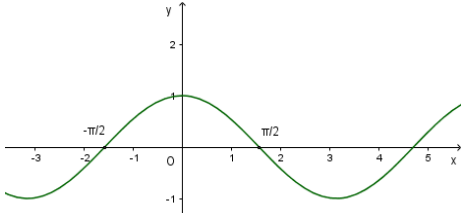
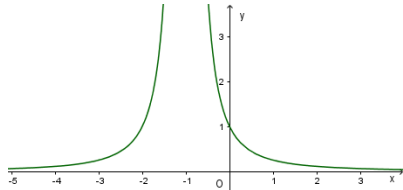
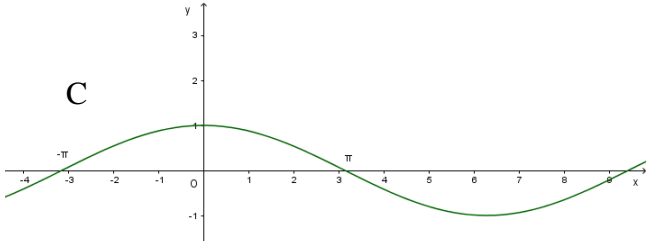
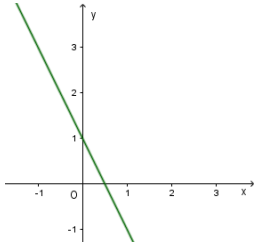
Like question 2, answer 3 also appears based on the shape of the two graphs.

As in the previous questions, the relationship between the variability of the function and its derivative is not mobilized by students on tasks involving reading graphs.

3.4. Analysis of question 4

Question 4: The first column is the algebraic expression of the functions $y = f(x)$. The second column is a graph of the derivative functions that are presented at random. Please show the graph of the derivative function corresponding to the given function.

Please answer by filling the corresponding letters in the box

The function $y = f(x)$	Graph of the function $y = f'(x)$
<p>1. $y = 2\sin \frac{x}{2}$</p>	<p>A</p> 
<p>2. $y = -x^2 + x$</p>	<p>B</p> 
<p>3. $y = \frac{2x+1}{x+1}$</p>	<p>C</p> 
	<p>D</p> 

Please answer by filling the corresponding letters in the box.

- Function 1 - Explain the reason:.....
- Function 2 - Explain the reason:.....
- Function 3 - Explain the reason:.....

Question 4 has a particular goal of putting the student in front of a type of task in which solving technique is started by finding the derivative by the rule. From the algebraic expression of the derivative, students have to work with graphs to find matching pairs of algebraic expressions and graphs.

On the graph, there is a special visible point. The amount of special points on each graph is just enough to use. A special feature makes it easy to connect the derivative expression to the graph of the derivative function. The graphs are given through at least one special point (0; 1). However, this point helps to reduce cost but it satisfies all derivative functions in the first column. It forces students to find more valuable points.

The result of the experiment shows that all 28 students who participated in the experiment recorded the choice to answer question 4. Of which, 25/28 students chose two answers A or C for function 1, nearly 90% of students chose one of these two answers to have a basis for us to judge that problems are using the graph function to identify the function to ensure the correct choice. Moreover, with function 1, answer A is the most chosen, this is not the correct answer, students choose this function and explain that $\frac{\pi}{2}$ is special value on that graph.

Students give the correct answer and explain how to implement the strategy to find special points on the derivative function graph and check on the derivative function to find the derivative function that satisfies that point. Thus, students have somewhat the relationship between the derivative expression and the derivative function graph when the shape of the derivative function graph has been identified from the derivative expression.

Besides, in function 2, five students after calculating the derivative of the function given by the algebraic expression in column 1 have identified the expression from which to know the graph of the derivative function.

Like the previous two questions, the student's misconception about the identity of the two original function graphs and the derivative function still appears.

Here, the student misinterprets the graph B as a parabola, but it also shows that the student has linked f to the quadratic function leading to the graph of the parabolic function.

For function 3, answer B is the most chosen, this is the correct answer. Students are interested in the domain of derivative functions.

When the question is given to the algebraic expression of the function is familiar to students, they have more strategies to solve.

4. Conclusion

Experiments allow us to test hypotheses, which show some consequences of the institutional relationship on students' relationship on the object of derivative functions:

- Student failed to mobilize the geometric meaning of the derivative, the relationship between the sign of the derivative and the variation of the function in tasks where a given graph representation is required;

- Facing tasks with graphical representation, students often build algebraic expressions and then derivative even if one strategy is more expensive than other strategies.

The failure of students (especially questions 1, 2 and 3) is not difficult to predict. However, in our opinion, these exercises suggest the types of additional tasks in teaching mathematics in Vietnam if we want to mention the meaning and application of derivatives and derivative functions. Also, the information gathered through experiments, especially with question 4, will help us guide the development of future teaching projects.

❖ **Conflict of Interest:** Author have no conflict of interest to declare.

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HÀM SỐ ĐẠO HÀM TRONG DẠY HỌC TOÁN BẬC TRUNG HỌC PHỔ THÔNG

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TÓM TẮT

Việc tính toán đạo hàm của một hàm số f cho kết quả là một biểu thức, nó cũng cho một hàm số. Hàm số đạo hàm f' và hàm số f có mối liên hệ qua lại nhìn từ phương diện đồ thị và biểu thức đại số. Bài báo chỉ ra mối liên hệ chưa đầy đủ giữa hàm số ban đầu và hàm số đạo hàm hiện diện trong chương trình và sách giáo khoa hiện hành môn Toán bậc trung học phổ thông Việt Nam. Từ đó gợi ý những kiểu nhiệm vụ mới có thể xuất hiện trong dạy học giải tích và phù hợp với kiểm tra đánh giá bằng trắc nghiệm khách quan.

Từ khóa: hàm số đạo hàm; đồ thị; dạy học giải tích