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Research Article

DESIGNING MATHEMATICAL MODELLING ACTIVITIES IN TEACHING VIEWPOINTS "APPROXIMATE x" OF THE CONCEPT OF LIMIT FUNCTION

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ABSTRACT

The paper summarizes the concepts related to modeling issues and design orientations for modeling activities in teaching mathematics. The meanings of the concept of limit of a function are also clarified in the next section of the paper. These meanings are not effectively mobilized by students when they learn to solve a situation beyond mathematics. Therefore, many mathematical modeling activities have been designed to help students comprehend and apply this concept to real-world problem-solving.

Keywords: Mathematical modeling; Mathematical modeling activity; the limit of a function; "approximate x"

1. Introduction

Mathematical modeling is a matter of concern, mentioned in many studies, such as Blum and Niss (1991), Kaiser (2007), Stillman (2010) and is the main topic of discussion of many of the world conferences. When applied to the general school, it has brought efficiency and demonstrates the importance of teaching and learning. For example, in Blum (1993)' s studies, Stillman (2010) shows that the mathematical model is essential to the students because it helps them equip their ability to be able to use the math in the form of inference, the study, creativity, solving the problem... In Vietnam, the mathematics curriculum aims to form and to develop students' mathematical competencies. Mathematics education in school will focus on training students to apply mathematics in real life, helping them realize the connection between math in schools with the real world. As noted above, we found that the application of mathematical modeling to teaching mathematics in Viet Nam is completely consistent with the world's education trends.

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In research about teaching and learning the concept of limit function at secondary high schools, Le (2011) has shown that students understand the concept of limit function through the symbol $\lim_{x \to a} f(x)$ is just the implementation of algebraic transformations to calculate limits, they do not understand the true meaning of the concept of limit function. Therefore, when educated research on assessing the ability to apply the concept of limitation to solving practical problems, Le and Pham (2017) have obtained poor results about student competency, they have difficulty in applying the concept of limit function to real-world problem-solving. Therefore, in this study, we expect that designing mathematical modeling activities in teaching the concept of limit function will help students form the competencies necessary for them can use the concept of limit function in situations that arise in life.

2. Content

2.1. Mathematical modeling process

According to Le (2014), "Mathematical modeling is a mathematical explanation for a non-mathematical system with specific questions that people ask on this system. The process of mathematical modeling is the process of setting up a mathematical model for a non-mathematical problem, solving problems in that model, then showing and evaluating solutions in real contexts, improving the model if the solution is not acceptable".

Figure below, Stewart (2012) illustrates the process of mathematical modeling. Given a real-world problem, our first task is to formulate a mathematical model by identifying and naming the independent and dependent variables and making assumptions that simplify the phenomenon enough to make it mathematically tractable.

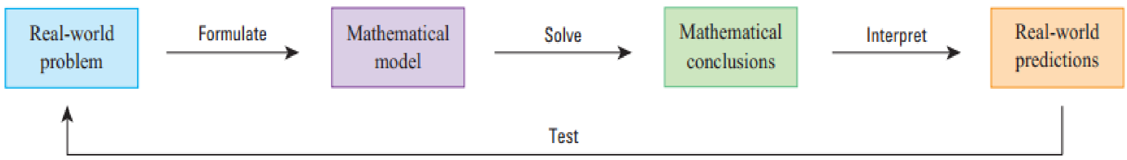

Figure 1. *Mathematical modeling process*

The second stage is to apply the mathematics that we know to the mathematical model that we have formulated to derive mathematical conclusions. Then, in the third stage, we take those mathematical conclusions and interpret them as information about the original real-world phenomenon by way of offering explanations or making predictions. The final step is to test our predictions by checking against new real data. If the predictions don't compare well with reality, we need to refine our model or to formulate a new model and start the cycle again.

2.2. Two perspectives on the concept of limit function in history

Talking about the views on the concept of limit function in history, Le (2011) said:

The first view of the concept of limit function existed from the Euclidean period (its ideas were expressed in the method of exhaustion) to Newton (1642-1727). He called this point of view "approximate x".

If a quantity x approaches a value of a (in the sense, it gets more and more close to a) then the quantity y - the dependent quantity x (a variable function x) - moves toward an L value. That is, x is getting closer and closer a then y is getting closer and closer to L.

A second view of the concept of limit function arises when Cauchy (1821) gives a precise definition of this concept. He called this point of view "approximate $f(x)$".

In the view of "approximate f(x)" we understand the concept of the limit expressed in today's modern notation $\lim_{x\to a} f(x) = L$ *which means that the approximation of f(x) with L that we expect will decide the approximation of* x *with* a *need to choose.*

The second view formed the right meaning of the concept of limit function. In 1876, Weierstrass expressed the view of "approximate $f(x)$" of the concept of limit function in the language ε, δ. This concise definition is still used at the university level today. With form language, one can express the term concept as follows:

$$\lim_{x\to a} f(x) = L \Leftrightarrow (\forall\varepsilon > 0, \exists\delta > 0 : |x-a| < \delta \Rightarrow |f(x)-L| < \varepsilon)$$

Although the first view does not represent the true essence of the concept of limit function, it gives us a visual view when approaching the concept. Therefore, when teaching the concept of limitation in high school, one does not overlook this view.

2.3. Designing mathematical modeling activities in teaching **"approximate x"** *of the concept of the limit function*

2.3.1. The orientation of designing mathematical modeling activities

According to Nguyen (2015), in teaching mathematics, teachers can design modeling activities as follows:

- Originating from practical problems to design activities, this situation must be suitable for students and contain mathematical knowledge that they have learned.
- Identifying a list of mathematical knowledge and skills that students need to build mathematical models and solve problems with mathematical tools.
- Creating links between real situations and maths: Making the situation clearer (idealizing, simplifying, specializing); Make appropriate assumptions; Identify the variables in a situation; Gather more real data for the situation; Describe in detail modeling situations.

2.3.2. Illustrate some activities

From the practical issues that appear in economics and biology, we choose two situations containing factors related to the limit of the function. These situations will be set up by our mathematical models for students to learn more about the situation and create conditions for the process of mathematical modeling to take place more favorably. Next, students are asked to work on the mathematical model that has been built through entrusting them to the question of forming an "approximation x" view of the concept of limit function. Hence, not only are students getting access to new knowledge but also trained in modeling competency, such as the computational capability to solve problems, ability to explain mathematical outcomes to the actual context.

Activity 1. Formulate for students the definition of a finite limit of a function from the view of "approximate x".

We choose a problem related to economics, students are required to estimate of expenses that a manager has to spend to operate a factory.

A manager who determines that when *x*% of his company's plant capacity is being used, the total cost of operation is *C* hundred thousand dollars, where

$$C(x)=\frac{8x^2-628x+4760}{x^2-58x-840}$$

The selected function has a rational function $\frac{u(x)}{v(x)}$ that usually appears when calculating limits in high school programs.

The domain D of this function is $D=\mathbb{R}\setminus\{-12;70\}$ and $\lim\limits_{x\to 70}C(x)=6.$

The large coefficients were chosen to guide students to the approximate calculation strategy using a calculator.

Question 1: *Estimate the operating cost of the plant when it operates at about 70% capacity.*

With this question, teachers and students will come to the conclusion: "We can't calculate the exact value of this function at $x=70$".

This conclusion raises the question: it is impossible to calculate the exact value C(70) but is it possible to approximate the value? If yes, how? If no, why not? "

The above problem is entrusted by question 2: *Calculate the value of the function* $C(x)$ *as x is closer to* 70.

We put the students in a situation where it would be easier to approximate the value of the function and thus the view of "approximate x" point is also implicitly formed.

Activity 2. Formulate for students the definition of a limit of a function at infinity from the view of "approximate x"

Biology is a very close subject, giving a practical case study about the time it takes a rat to walk through a lab labyrinth is a situation containing mathematical elements. Each time the mouse passes through the maze will correspond to a specified period, the relationship between the number of times and the time it passes through its maze denotes a function. The choice of a specific function denoting the aforementioned relationship is more conducive to students' access to the concept of limit function at infinity.

Students are entrusted with the following question:

*To study the rate at which animals learn, a psychology student performed an experiment in which a rat was sent repeatedly through a laboratory maze. Suppose the time required for the rat to traverse the maze on the n*th *trial was approximately* $T(n)=\frac{5n+16}{n}$ *minutes. What happens to the time of traverse as the number of trials n increases indefinitely? Interpret your result.*

The above problem requires students to test the time the rat goes through the maze in some specific cases to make their predictions, their predictions will be more accurate if they calculate the value of the function when giving n greater. In some cases, teachers can ask additional questions as a suggestion for students if they have difficulty.

Question: *Calculate the values of the function T (n) when* $n=10^3; n=10^5,...$

3. Results and discussion

With two designed activities, the meaning "approximate x" of the concept of limit function will have a chance to appear and come into students' awareness. When conducting two practical classroom activities, teachers can base on the learning environment and the ability of students to be able to add some additional questions to the successful implementation of goals in this pedagogy.

4. Conclusion

The essence of the concept of limit function has been deeply expressed through mathematical modeling activities. The student's knowledge challenges in understanding this abstract concept are somewhat reduced. Through the process of solving modeling activities, students are trained in mathematical modeling competencies: the ability to use knowledge to solve mathematical problems, the ability to use mathematical conclusions into explaining them about reality.

❖ ***Conflict of Interest:*** *Author have no conflict of interest to declare.*

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THIẾT KẾ HOẠT ĐỘNG MÔ HÌNH HÓA TOÁN HỌC TRONG DẠY HỌC QUAN ĐIỂM "XẤP XỈ X" CỦA KHÁI NIỆM GIỚI HẠN HÀM SỐ

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TÓM TẮT

Bài báo giới thiệu tóm lược các khái niệm có liên quan đến vấn đề mô hình hóa và các định hướng thiết kế các hoạt động mô hình hóa trong dạy học Toán. Các nghĩa của khái niệm giới hạn hàm số cũng được làm rõ trong phần tiếp theo của bài báo. Trong thực tế, những nghĩa đó không được học sinh huy động hiệu quả khi học đứng trước việc giải quyết một tình huống ngoài toán học. Do đó, một số hoạt động mô hình hóa toán học đã được chúng tôi thiết kế với mục đích giúp học sinh lĩnh hội và vận dụng khái niệm này vào giải quyết các vấn đề trong thực tế.

Từ khóa: mô hình hóa toán học; hoạt động mô hình hóa toán học; giới hạn của hàm số; "xấp xỉ x"