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**Research Article**

# EVALUATING NUCLEAR RADIATIVE STRENGTH FUNCTION MODELS BASED ON THE EXPERIMENTAL NEUTRON-CAPTURE CROSS-SECTION OF $^{55}Mn(n,\gamma)^{56}Mn$ REACTION

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**ABSTRACT**

*Describing the nuclear radiative strength function (RSF) at energies below the neutron separation energy ($B_n$) is crucial for providing reliable input in nuclear reaction and nuclear astrophysics calculations. In this study, we evaluate eight RSF models, encompassing both phenomenological and microscopic approaches, by employing them as input to calculate the neutron-capture cross-section of the $^{55}Mn(n,\gamma)^{56}Mn$ reaction. The result is then compared with the experimental one. The results indicate that microscopic RSF models built on the Hartree-Fock mean field theory offer good descriptions of the cross-section, with notable performance observed in the temperature-dependent Hartree-Fock-Bogoliubov (T-dependent HFB) model. Selecting such appropriate RSF models ensures reliable input for calculations related to nuclear reactions and astrophysics.*

***Keywords:*** Hauser-Feshbach statistical theory, (n,γ) reaction; neutron-capture cross-section; radiative strength function

## 1. Introduction

The gamma or radiative strength function (RSF) is defined as the average probability of electromagnetic transitions per unit of gamma-ray energy $E_\gamma$ (Blatt & Weisskopf, 1952). This quantity holds a significant role in nuclear physics, finding utility in studies spanning from nuclear structure to nuclear astrophysics, particularly playing a pivotal role in nuclear reactions. During reactions, nuclei undergo excitation or de-excitation by absorbing or emitting energy in various forms. Particularly, the absorption or emission of photons within the energy range of gamma rays is commonly employed to study reaction properties. In the case of neutron capture-gamma emission reactions, a nucleus absorbs energy through

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interaction with an incident neutron, capturing it to form an excited compound nucleus. Subsequently, this nucleus emits gamma rays to return to its ground state. RSF is the quantity utilized to describe such excitation and de-excitation processes (Goriely et al., 2019). Together with another crucial quantity, the nuclear level density (NLD), RSF provides insight into the characteristic properties of the reaction, such as reaction cross-section or reaction rate.

RSF is typically extracted in two energy regions, separated by the neutron separation energy $B_n$. In the region above Bn, there exists a considerable amount of experimental data extracted through giant dipole resonances (GDR) via (γ,n) reactions. However, before 2000, experimental RSF data in the energy region below $B_n$ were scarce. It was not until the introduction of the Oslo method in 2000 that the extraction of RSF in the low-energy region significantly improved, utilizing the light ions induced or inelastic-scattering reactions (particle,γ), leading to an increase in experimental RSF data in this region year by year (see Oslo database). Thus, theoretical models describing RSF in $E_\gamma < B_n$ are necessary due to the lack of experimental data. Some phenomenological RSF models are Kopecky-Uhl generalized Lorentzian (GLO) (Kopecky & Uhl, 1990; Kopecky et al., 1993), Brink-Axel Lorentzian or Standard Lorentzian (SLO) (Brink, 1957; Axel, 1962), Hartree-Fock Bardeen–Cooper–Schrieffe (BCS) approach (Goriely & Khan, 2002), Hartree-Fock-Bogolyubov (HFB) approach (Goriely & Khan, 2004), Goriely's hybrid model (Goriely, 1998), Goriely T-dependent HFB (Hilaire et al., 2012), T-dependent relativistic mean field (RMF) (Arteaga & Ring, 2008), and Gogny D1M HFB plus quasiparticle-random-phase approximation (QRPA) (Martini et al., 2014). These theoretical models are founded on a classic assumption known as the Brink-Axel hypothesis (Brink, 1955; Axel, 1962), which posits that the RSF solely depends on the emitted gamma energy $E_\gamma$ and is independent of the excitation energy E* of the nucleus, or in other words, it is temperature-independent. However, a recent microscopic model that concurrently describes the RSF and NLD has revealed that the temperature dependence of RSF, manifested through damping of the giant dipole resonance (GDR), challenges the validity of the Brink-Axel hypothesis (Hung et al., 2017).

In this study, the RSF was semi-empirically extracted by leveraging experimental neutron-capture cross-section data and theoretical datasets of default RSF and NLD in the Talys code (Koning et al., 2007) via the Hauser-Feshbach statistical theory (Hauser & Feshbach, 1952). The default RSF and NLD models serve as input in cross-section calculations, allowing comparison with experimental data. Subsequently, the most suitable RSF model was selected based on this comparison. This approach offers a means to select an appropriate RSF model for reaction theory calculations without requiring experimental RSF data. Additionally, this method leverages the richer experimental data available for cross-sections, facilitating its implementation. The $^{55}$Mn(n,γ)$^{56}$Mn reaction is used as a typical candidate in this work.

## 2. Methodology

### *2.1. RSF theoretical models*

In RSF modeling, theoretical approaches typically fall into two categories: phenomenological models and microscopic models. This study employs both types of models to compute the cross-section. Phenomenological models, such as RSF-1 (Kopecky & Uhl, 1990; Kopecky et al., 1993), RSF-2 (Brink, 1957; Axel, 1962), and RSF-5 (Goriely, 1998), utilize inputs like GDR parameters - such as peak energy, width, and cross-section - derived from experiments or theoretical models. Some models took into account a constant temperature, which is predicted in the Kadmenskii-Markushev-Furman (KMF) model (Kadmenskii et al., 1983). These RSF predictions are commonly regarded as fitting functions.

***Table 1.*** *Various gamma-ray strength function models available in the Talys code*

| Model | Model no | Reference |
|---|---|---|
| Kopecky-Uhl generalized Lorentzian | RSF-1 | Kopecky and Uhl, 1990; Kopecky et al, 1993 |
| Brink-Axel Lorentzian | RSF-2 | Brink, 1957; Axel, 1962 |
| Hartree-Fock-BCS | RSF-3 | Goriely and Khan, 2002 |
| Hartree-Fock-Bogolyubov | RSF-4 | Goriely and Khan, 2004 |
| Goriely's hybrid model | RSF-5 | Goriely, 1998 |
| Goriely T-dependent HFB | RSF-6 | Hilaire et al., 2012 |
| T-dependent RMF | RSF-7 | Arteaga and Ring, 2008 |
| Gogny D1M HFB+QRPA | RSF-8 | Martini et al., 2014 |

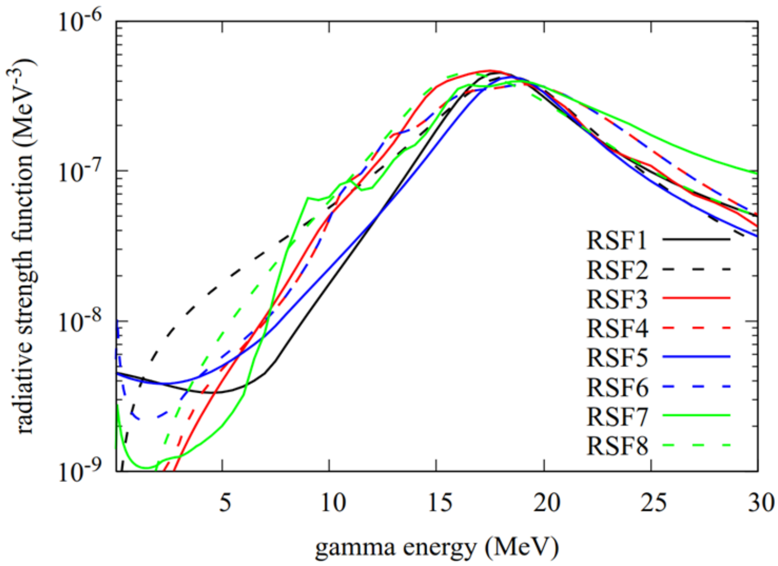


***Figure 1.*** *Different RSF models obtained from the Talys code*

Additionally, expressions such as Lorentzian, Breit-Wigner, or Gaussian are commonly employed to describe RSF in theoretical models (Goriely et al., 2019 ). For microscopic RSF models, the input parameters used to describe the RSF are directly calculated from the model. These models typically involve the Hartree-Fock mean field, accounting for pairing effects, such as Hartree-Fock-BCS (RSF-3) or Hartree-Fock-Bogoliubov (RSF-4). Additionally, some models consider temperature effects, such as T-dependent HFB (RSF-6) or T-dependent RMF (RSF-7), as well as more complex microscopic models like Gogny D1M HFB+QRPA (RSF-8). These models are all pre-set as options in the Talys code (Koning et al., 2007), which stands as the most widely used program for cross-section calculations in the global nuclear physics community. Table 1 provides a list of these models, while Figure 1 presents the RSF predictions derived from these models.

Phenomenological models serve primarily as fitting functions and may lack a clear representation of the underlying physical mechanisms. Conversely, microscopic models provide insights into the physical nature of the phenomena. Particularly, microscopic models can naturally elucidate specific resonances in the low-energy region, such as Pygmy dipole resonances, scissor resonances, or enhancements in the very low-energy region (upbend) (Hung et al., 2017; Martini et al., 2014; Schwengner et al., 2017).

### *2.2. Evaluation methods*

Based on the Hauser-Feshbach statistical theory, the calculation of neutron-capture cross-sections relies on various inputs such as NLD and RSF. The objective is to ensure that the calculated cross-sections align with experimental data by selecting appropriate models for NLD and RSF. To evaluate the compatibility of various RSF models for the $^{56}$Mn nucleus, each model listed in Table 1 was individually implemented in the TALYS v1.95 program to compute the cross-section for the $^{55}$Mn(n,$\gamma$)$^{56}$Mn reaction, which is then compared with experimental data. In our Talys calculations, all inputs, except RSF, remain at their default values to maintain consistency. The computation of neutron-capture cross-sections depends on several factors, including NLD, discrete levels in the low-energy region, and masses of target and compound nuclei. Notably, NLD exerts the most significant influence, while other parameters have minimal effects, with detailed recommendations available in the RIPL-2 and RIPL-3 nuclear databases (see RIPL-2 and RIPL-3). Specifically, the temperature - dependent Hartree-Fock-Bogolyubov plus combinatorial method (HFBT) (Hilaire et al, 2012) is employed in our calculations for NLD of $^{55}$Mn(n,$\gamma$)$^{56}$Mn, as it predicts an average level spacing $D_0$ of 2301.47 eV, closely matching experimental data within the range of 2300±400 eV (RIPL-3). Among the default Talys models, the HFBT offers the best prediction for $D_0$. Furthermore, fixing all inputs enables us to discern the influential role of RSF on the calculated cross-sections.

## 3. Results and discussions

Figure 2 shows the calculated cross-sections of $^{55}Mn(n,\gamma)^{56}Mn$ reaction from eight RSF models compared with experimental data sourced from Garg1978 (Garg et al., 1978), Dovbenko1969 (Dovbenko et al, 1969), Stupegia1968 (Stupegia et al., 1968), and Menlove1967 (Menlove et al., 1967). Most theoretical models predict quite well the data above 10 MeV. It is evident that RSF-2 and RSF-8 models substantially overestimate the experimental data within the energy range of 0 - 10 MeV, whereas the RSF-1 and RSF-7 tend to underestimate it. Among them, RSF-7 describes quite well a part of the experimental data in the region larger than 1 MeV. The RSF-3 to RSF-6 generally exhibit a relatively good fit with the experimental data, with RSF-3, RSF-4, and RSF-6 (all are microscopic models) performing the best in the whole energy region, and RSF-5 underestimating the data in $E_\gamma > 1$ MeV. Notably, no phenomenological model describes the RSF for the entire energy range as well as the microscopic models.

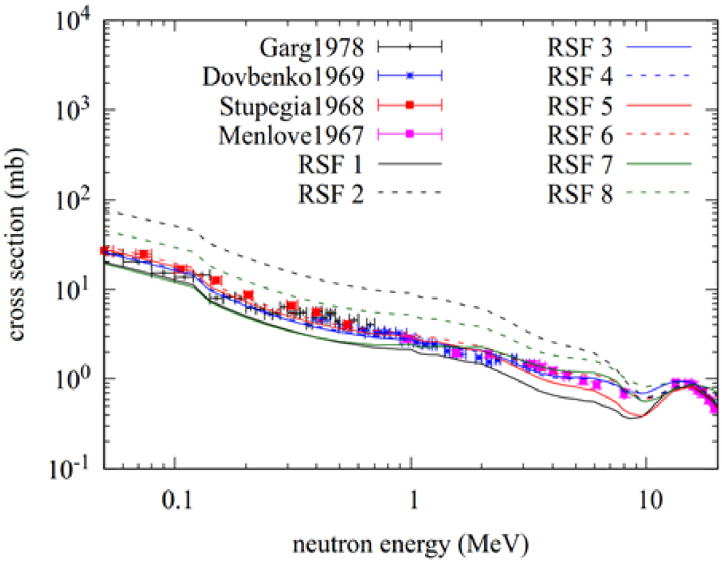


***Figure 2.*** *Comparison of calculated cross-sections of $^{55}Mn(n,\gamma)^{56}Mn$ reaction corresponding to different RSF models*

To determine the most appropriate RSF model among the three promising candidates, RSF-3, RSF-4, and RSF-6, we evaluate the total radial gamma width derived from all available models. Figure 3 shows the calculated values of total radiative gamma width obtained with various RSF models. Given that the total radiative gamma width serves as a crucial metric frequently employed to gauge nuclear theoretical models, the ideal RSF models should yield width values consistent with experimental data.

From Figure 3, it is evident that the microscopic RSF models yield width values remarkably consistent with experimental data (RIPL-3), with the RSF-6 model precisely matching the experimental value. This underscores the superiority of the RSF-6 model, i.e., the temperature-dependent Hartree-Fock-Bogoliubov (HFB) model, in describing the RSF of the $^{56}Mn$ compound nucleus. This result is logical considering that the HFB model itself is a dependable microscopic approach widely employed in nuclear structure calculations that take into account the pairing effect. Moreover, the prediction of RSF as temperature-dependent aligns well with its nature and raises a question regarding the validation of the Brink-Axel (Hung et al., 2017) hypothesis. Hence, the utilization of the temperature-dependent HFB model proves highly suitable for microscopically calculating RSF.

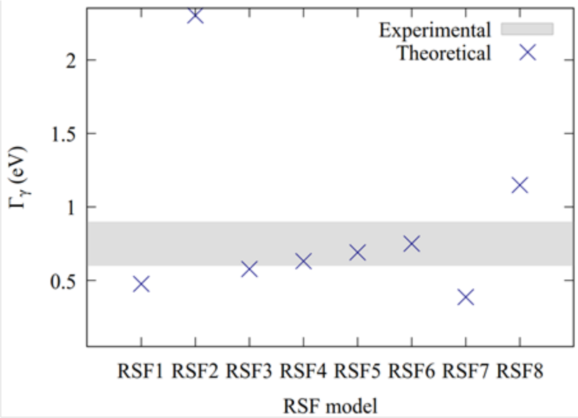


***Figure 3.*** *The total radiative gamma widths of $^{55}Mn(n,\gamma)^{56}Mn$ reaction obtained from different RSF models*

In short, employing microscopic models for NLD and RSF to characterize neutron-capture cross-sections of compound nuclear reactions is appropriate due to their ability to describe the physics properties of the nuclear system at a microscopic level. In this study, the combination of the HFBT model for NLD and the temperature-dependent HFB model for RSF yielded an excellent description of the cross-section for the $^{55}Mn(n,\gamma)^{56}Mn$ reaction. The identification of such NLD and RSF models as input will increase the reliability of further calculations in nuclear reactions or nuclear astrophysics.

## 4. Conclusions

In this study, we computed the neutron-capture cross-section of the $^{55}Mn(n,\gamma)^{56}Mn$ reaction by employing eight RSF models within the framework of the Hauser-Feshbach statistical theory. Through a comparison of the obtained results with experimental data, we assessed the suitability of the employed RSF models. The analysis reveals that microscopic RSF models (RSF-3, RSF-4, and RSF-6) offer the most reliable inputs for cross-section calculations, as they consistently yield results that align with experimental data. In particular, the combination of the HFBT model for NLD and the T-dependent HFB model for RSF (RSF-6) proves especially effective in characterizing the $^{55}Mn(n,\gamma)^{56}Mn$ reaction.

❖ ***Conflict of Interest:*** *Authors have no conflict of interest to declare.*

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## ĐÁNH GIÁ CÁC MÔ HÌNH HÀM LỰC BỨC XẠ HẠT NHÂN DỰA TRÊN TIẾT DIỆN BẮT NEUTRON THỰC NGHIỆM CỦA PHẢN ỨNG $^{55}$Mn(n,γ)$^{56}$Mn

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**TÓM TẮT**

*Mô tả hàm lực bức xạ hạt nhân (RSF) ở năng lượng dưới năng lượng tách hạt neutron ($B_n$) là việc cần thiết để cung cấp đầu vào đáng tin cậy trong các tính toán phản ứng hạt nhân và thiên văn học hạt nhân. Trong nghiên cứu này, chúng tôi đánh giá tám mô hình RSF, bao gồm cả các mô hình hiện tượng luận và mô hình vi mô, bằng cách sử dụng chúng như đầu vào để tính toán tiết diện bắt neutron của phản ứng $^{55}$Mn(n,γ)$^{56}$Mn. Kết quả tính toán sau đó được so sánh với dữ liệu thực nghiệm. Kết quả của chúng tôi cho thấy rằng các mô hình RSF vi mô được xây dựng trên lí thuyết trường trung bình Hartree-Fock mô tả tốt tiết diện phản ứng, đặc biệt là đối với mô hình Hartree-Fock-Bogoliubov phụ thuộc vào nhiệt độ (T-dependent HFB). Việc lựa chọn các mô hình RSF phù hợp như vậy đảm bảo đầu vào đáng tin cậy cho các tính toán liên quan đến các phản ứng hạt nhân và thiên văn học.*

***Từ khóa:*** lí thuyết thống kê Hauser-Feshbach; phản ứng (n,γ); tiết diện bắt nơ tron; hàm lực bức xạ