TẠP CHÍ KHOA HỌC
TRƯỜNG ĐẠI HỌC SƯ PHẠM TP HỒ CHÍ MINH
Tập 23, Số 3 (2026): 631-643
Website: https://journal.hcmue.edu.vn
ISSN: 2734-9918

HO CHI MINH CITY UNIVERSITY OF EDUCATION
JOURNAL OF SCIENCE
Vol. 23, No. 3 (2026): 631-643
https://doi.org/10.54607/hcmue.js.23.3.5099(2026)

Research Article

DETERMINATION OF RADIOACTIVITY BASED ON UNFOLDED GAMMA SPECTRA USING THE GRAVEL METHOD AND RESPONSE MATRIX FROM MCNP SIMULATION

Le Quang Vuong[1,2*], Nguyen Huynh Khanh Duyen[1], Nguyen Huynh Duy Khang[1], Hoang Duc Tam[1], Phan Long Ho[2,3], Tran Thien Thanh[2], Chau Van Tao[2]

[1]Ho Chi Minh City University of Education, Vietnam
[2]University of Science, Vietnam National University Ho Chi Minh City, Vietnam
[3]Institute of Public Health in Ho Chi Minh City, Ho Chi Minh City, Vietnam
[*]Corresponding author: Le Quang Vuong – Email: vuonglq@hcmue.edu.vn
Received: June 07, 2025; Revised: October 31, 2025; Accepted: November 18, 2025

ABSTRACT

This study presents an improved method for gamma-spectrometry analysis, particularly in low-energy regions, by combining the GRAVEL unfolding algorithm with a high-fidelity detector response matrix simulated using the MCNP6 code. The response matrix was constructed for a p-type HPGe detector up to 1 MeV. The method underwent a rigorous validation process using both synthetic spectra and a certified multi-nuclide point source, providing a strong foundation for its reliability. The results demonstrate a significant enhancement in peak separation and activity quantification, with relative deviations from certified activities reduced from as high as 47.8% (direct analysis) to below 4.3% after unfolding. This approach proves highly effective for accurately measuring low-energy and weak gamma emitters.

Keywords: FEP efficiency; Gamma Spectrometry; MCNP; Radioactivity; Unfolding

1. Introduction

Measuring natural radioactivity is essential for ensuring radiation safety and protecting the environment (Byon et al., 2022). Gamma spectrometry with HPGe detectors is widely used due to its simplicity in sample preparation, reasonable cost, and most importantly, its capability for simultaneous multi-radionuclide analysis. However, it faces challenges in measurement accuracy due to spectral distortion caused by Compton scattering and the lack of prior information on the incident radiation, particularly for low-energy or weak gamma emitters. These factors elevate background and obscure photopeaks. Moreover, the thick dead layer in p-type HPGe detectors reduces sensitivity to low-energy photons, such as the 59.5 keV gamma rays from ^{241}Am (Dryak et al., 2010).

Cite this article as: Le, Q. V., Nguyen, H. D. K., Hoang, D. T., Phan, L. H., Tran, T. T., & Chau, V. T. (2026). Determination of radioactivity based on unfolded gamma spectra using the Gravel method and response matrix from MCNP simulation. *Ho Chi Minh City University of Education Journal of Science 23*(3), 631-643. https://doi.org/10.54607/hcmue.js.23.3.5099(2026)

To enhance the accuracy of activity determination, gamma spectrum deconvolution techniques utilising detector response functions have been developed to separate overlapping peaks and effectively enhance spectral resolution (Li et al., 2019a). Various algorithms have been applied, including the least squares method (Li et al., 2019b), Maximum Likelihood Expectation Maximisation (ML-EM) (Liu et al., 2020), Maximum Entropy Method (MEM) (Baré & Tondeur, 2011), and the Gold algorithm (He et al., 2016; Loan et al., 2016). The UMG software package (PTB, 2004), renowned for its robustness and flexibility, incorporates the GRAVEL algorithm, an enhanced version of the Gold variant. GRAVEL, with its proven effectiveness in handling overlapped spectra, instils confidence in its performance. It supports multiple detectors, including HPGe, NaI(Tl), $CeBr_3$, and LSC, and its value in reliable gamma spectrum analysis has been highlighted in recent studies (Dombrowski, 2024; Li et al., 2020). Although these algorithms are well known, their practical application in the low-energy region (<100 keV) remains challenging. This study focuses on developing and validating a comprehensive procedure that combines detailed MCNP simulations with the GRAVEL algorithm to address this issue.

In this study, we propose an improved method for accurately determining the activity of radionuclides from gamma spectra using deconvolution techniques. Specifically, the UMG program, combined with the GRAVEL algorithm, is applied to process gamma spectra at low energies, including 59.5 keV (^{241}Am), which is often affected by background-related errors. To ensure the reliability of the deconvolution results, we construct the response matrix using Monte Carlo simulations with the MCNP6 code, a method known for its accuracy and precision in modelling radiation - detector interactions. Finally, we validate the model using experimental gamma spectra from a point source containing multiple radionuclides, thereby demonstrating the reliability and practical applicability of the proposed method.

2. Materials and methods

2.1. Experimental measurements

Gamma spectra measurements were performed using a p-type HPGe detector (supplied by ORTEC, model GEM50P4-83) (Tran et al., 2018). At an energy of 1.33 MeV (^{60}Co), this HPGe detector has an energy resolution (FWHM) of 1.9 keV and a relative efficiency of 50%. The gamma spectra were recorded using Maestro software (Maestro-32, 2003) with an energy range of up to 3 MeV, corresponding to 16,384 energy channels. The peak counts at the energies of interest were analyzed using Colegram software (Ménesguen & Lépy, 2021).

The gamma-emitting standard source is a blend of several radionuclides with an energy range from 46.5 to 1836.1 keV. The source, purchased from Eckert & Ziegler Isotope Products, comes with a combined uncertainty of approximately 3% (product code 7503, (Eckert & Ziegler)). The source has a cylindrical geometry with an outer diameter of 21 mm and a height of 6.35 mm. Its outer casing is made of polymethyl methacrylate (PMMA). At the centre of the source is a cavity containing the radioactive material, with a diameter of 5 mm and a height of 3 mm. The distance from the cavity surface to the base of the source is

2.95 mm. The source is placed on a plastic tray such that its centre is aligned coaxially with the detector. The distance from the detector window to the base of the source is 10 cm. At this distance, the total coincidence summing effect of gamma rays emitted from cascade-emitting radionuclides can be neglected (Le et al., 2019). The gamma spectrum was recorded over 86,400 seconds.

2.2. Monte Carlo simulations

Monte Carlo simulations using MCNP6.1 (Pelowitz, 2013) were conducted with meticulous attention to detail to model radiation transport in a low-background gamma spectrometry system. Figure 1 illustrates the simulated geometric configuration for measuring a multi-radionuclide point standard source at a distance of 10 cm. The geometry and materials of the HPGe detector, shielding, and source setup were accurately reproduced to match the experimental results. Mode P simulated photon interactions (photoelectric, Compton, Rayleigh, pair production, fluorescence). The F8 tally, available in MCNP6, was employed to record the pulse height spectrum (PHS) over an energy range of 6×10^{-4} to 2.798 MeV, corresponding to 16,384 energy channels. Simulations tracked 4×10^{8} to 10^{9} particle histories to maintain statistical uncertainty below 0.5%. These results were employed to construct the detector response function and assess HPGe detection efficiency.

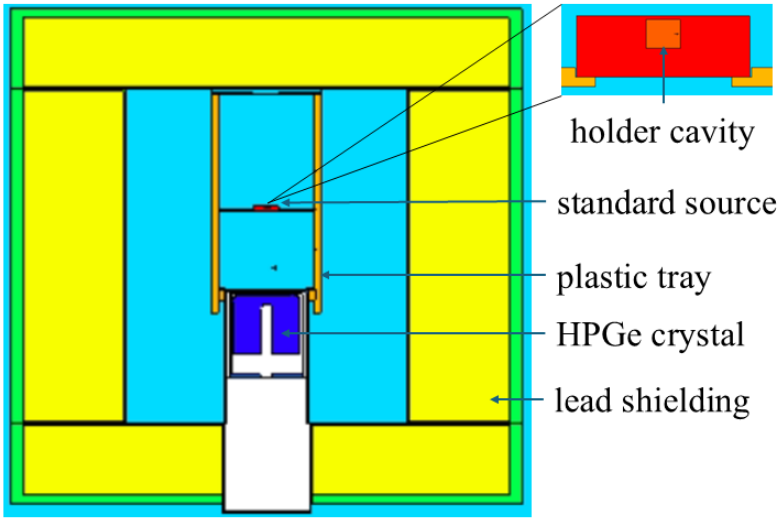

Figure 1. *MCNP6 simulation of the gamma-ray spectrum measurement*

2.3. Short description of the GRAVEL algorithm

The objective of the spectrum unfolding process is to reconstruct the undistorted spectrum $f(E)$. For the gamma spectrometry method, the detector's pulse height spectrum is defined by the Fredholm integral equation (Chen et al., 2014):

$$\frac{dN}{dH} = \int R(H,E)\, f(E) dE \tag{1}$$

Where dN/dH is the number of recorded events at pulse height H as measured by the detection system, and $R(H,E)$ is the detector response matrix. This function $f(E)$ describes

the probability that radiation with energy *E* is detected at a channel corresponding to pulse height *H*. In practice, the signal distribution can only be recorded in discrete channels. Therefore, equation (1) is expressed in the discrete form as:

$$\vec{N} = \vec{R} \cdot \vec{f} \tag{2}$$

In which $\vec{N}$, $\vec{R}$, and $\vec{f}$ are the vectors of the measured gamma spectrum by the detector, the response matrix, and the energy distribution spectrum of the source, respectively. Thus, the count at channel *i* of the detector is determined by the following formula:

$$N_i = \sum_j R_{ij} f_j \tag{3}$$

In which R_{ij} is the element of the response matrix, N_i (with $\mathrm{i} = 1, 2, ..., \mathrm{n}$) is the recorded count at the *i*-th channel, and f_j (with $\mathrm{j} = 1, 2, ..., \mathrm{m}$) is the actual value of the measured spectrum in the *j*-th energy interval.

The GRAVEL iterative algorithm (GRV_MC33, (PTB, 2004)) was used to solve the experimental spectrum via the response matrix by maximizing relative entropy under measurement constraints. GRV_MC33 was developed explicitly for multi-channel data and significantly improves upon the original GRAVEL version from the HERRO package. Input/output formats were modified for integration with the HEPRO package. These features make GRV_MC33 a powerful and practical tool for spectrum deconvolution and activity determination in gamma spectrometry. The basic formulas for the iterative process according to the GRAVEL algorithm implemented in the GRV_MC33 program are as follows (Dombrowski, 2024):

$$\begin{aligned} f_i^{J+1} &= f_i^J \cdot \exp\left[\left(\sum_k W_{ik}^J \cdot \log\left(N_k / \sum_{i'} R_{ki'} f_{i'}^J\right)\right) / \sum_k W_{ik}^J\right] \\ &= f_i^J \cdot \exp\left[\left(\sum_k W_{ik}^J \cdot \log\left(N_k / T_k^J\right)\right) / \sum_k W_{ik}^J\right] \end{aligned} \tag{4}$$

Where $W_{ik}^J = \dfrac{R_{ki} f_i^J}{\sum_{i'} R_{ki'} f_{i'}^J}\left(\dfrac{N_k^2}{\sigma_k^2}\right) = \dfrac{T_{ki}^J}{T_k^J}\left(\dfrac{N_k^2}{\sigma_k^2}\right)$, $T_k^J = \sum_i R_{ki} f_i^J$, $T_{ki}^J = R_{ki} f_i^J$. The variables in the equation are interpreted as follows (each element represents the number of counts in a spectral bin): f_i^J is the *i*-th element of the unfolded spectrum at the *j*-th iteration; R_{ki} is an element of the response matrix; N_k is the *k*-th bin of the measured spectrum; σ_k is the estimated error of the measurement value at channel k; W_{ik}^J is an element of the normalisation matrix; T_k^J is the *k*-th bin of the test spectrum; T_{ki}^J is the contribution of energy *i* to bin *k* in the test spectrum; $k \in \{1, 2, 3, ..., m\}$ and $i \in \{1, 2, 3, ..., n\}$.

However, the measured spectrum always contains uncertainties due to statistical fluctuations, and the iterative process may, in principle, continue indefinitely (since a perfect

match between the measured and the reconstructed spectrum can never be achieved). Therefore, a criterion is needed to determine the optimal number of iterations. A helpful indicator at each iteration J is the parameter χ^J, defined as follows:

$$\left(\chi^J\right)^2 = \sum_k \left(\frac{N_k - \sum_i R_{ki} f_i^J}{\sigma \cdot \sqrt{F}} \right)^2 = \sum_k \left(\frac{N_k - T_k^J}{\sigma \cdot \sqrt{F}} \right)^2 \quad (5)$$

The parameter F represents the number of degrees of freedom, corresponding to the number of spectral channels being unfolded. In many cases, the value of $\left(\chi^J\right)^2$ will reach a minimum, or decrease below a user-defined threshold value ($\chi^2_{\text{threshold}}$).

With a non-negative default spectrum, this iterative procedure always yields a non-negative solution spectrum and tends to achieve a lower chi-square value in subsequent iterations. For formula (4) to make sense, $N_k \neq 0$ and $T_k^J \neq 0$ must be specified. In this study, the zero value at the measurement channel will be replaced with a value of 10^{-20}. To run the program, three input files need to be prepared: *(i) Measured spectrum file:* contains the count values for each channel; *(ii) Response function matrix file:* generated from MCNP simulations; *(iii) Default spectrum file:* estimates the starting spectrum for the unfolding process. In this study, the initial spectrum file was generated from an MCNP simulated spectrum with a single-energy peak to be unfolded.

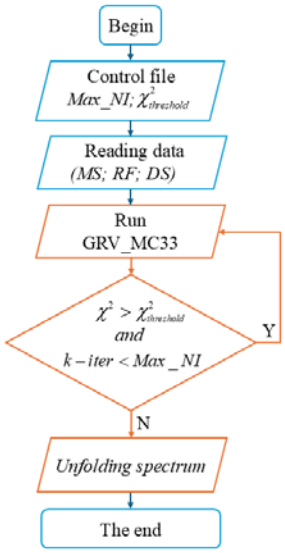

Figure 2. *Diagram of the steps for gamma spectrum unfolding using GR_MC33*

The UMG manual provides detailed parameter descriptions for each file (PTB, 2004). Figure 2 illustrates the steps for running the GRV_MC33 program. The unfolding process begins with reading the control file, which includes key parameters such as the maximum number of iterations (Max_NI) and the chi-square threshold ($\chi^2_{\text{threshold}}$), both of which serve as stopping criteria. The program then loads the measured spectrum (MS), the detector

response matrix (RS), and the default spectrum (DS), all of which are essential for the unfolding process. GRV_MC33 performs iterative unfolding by checking whether the current iteration count (*k-iter*) is below the maximum (Max_NI) and whether the chi-square value (χ^2) remains above the threshold ($\chi^2_{\text{threshold}}$). If both conditions are satisfied, the iteration continues; otherwise, the final unfolded spectrum is produced. In this study, we conducted a series of tests by varying the number of iterations from 10 to 10,000 to determine the corresponding stable chi-square value. Based on these results, we selected the optimal number of iterations and chi-square threshold, and then used these parameters for the final unfolding run.

The T-test spectrum is calculated by multiplying the solved spectrum f by the response matrix *R* and comparing it with the measured spectrum. If the test and measured spectra match, the logarithmic term f_i in the GRAVEL formula becomes zero, halting further updates, indicating convergence. Additionally, the normalisation matrix *W* is derived from the response matrix *R*. Each column of *R*, weighted by the solved spectrum, is normalised to a value of 1, ensuring consistency.

3. Results and discussion

3.1. Calculation of full energy peak efficiency and the response function

Simulations using MCNP6 up to 10^9 histories ensured statistical uncertainties below 0.01% at photopeak energies. The results show a significant agreement in both the spectral peak shapes and the full width at half maximum (FWHM) between the simulated and experimental spectra at medium and high energies, particularly for the photopeaks of ^{137}Cs (661.7 keV), ^{60}Co (1173.2, 1332.5 keV), and ^{88}Y (898.0 keV, 1836.1 keV) (see Figure S1, supplementary). However, at lower energies (see Figure S2), such as those of ^{210}Pb (46.5 keV), ^{241}Am (59.5 keV), ^{109}Cd (88.0 keV), and ^{57}Co (122.1, 136.5 keV), the experimental spectra exhibit a higher Compton background compared to the simulations. This is due to the use of a multi-radionuclide source, which resulted in spectral overlaps from multiple emissions, thereby increasing the background. In contrast, simulations employed monoenergetic sources, eliminating overlap and reducing Compton background.

The simulated and experimental full energy peak efficiency (FEPE) were determined using equations (6) and (7):

$$\varepsilon_{MC}(E) = \frac{N_{MC}}{\Omega_{MC}^{total}} \tag{6}$$

$$\varepsilon_{Exp}(E) = \frac{N_P(E)}{A_c \times I_\gamma \times t} \tag{7}$$

The simulated efficiency $\varepsilon_{MC}(E)$ is calculated as the ratio of the number of events that deposit their entire energy within the active volume of the detector (Ω_{MC}^{total}) to the total number of simulated photons (N_{MC}) at a given energy E (keV). The experimental efficiency $\varepsilon_{Exp}(E)$ is determined based on the net peak area $N_P(E)$, the certified activity $A_c(Bq)$ of the source,

the photon emission intensity $I_\gamma(\%)$, and the live acquisition time t. The measurement uncertainty of FEPE includes uncertainties in the peak counts, the source activity, and the gamma emission intensity, and is evaluated using standard uncertainty propagation methods (GUM, 2008). Figure 3a compares experimental and simulated FEPE values using MCNP6, showing excellent agreement (within a 2% deviation) from 661.7 keV to 1836.1 keV, which validates the accuracy and reliability of the simulation under experimental conditions. Figure 3b presents the simulated FEPE for a point source placed 10 cm from the detector, across a wide energy range from 46.5 keV to 1836.1 keV. These results confirm the model's precision and applicability for efficiency calibration in gamma spectrometry.

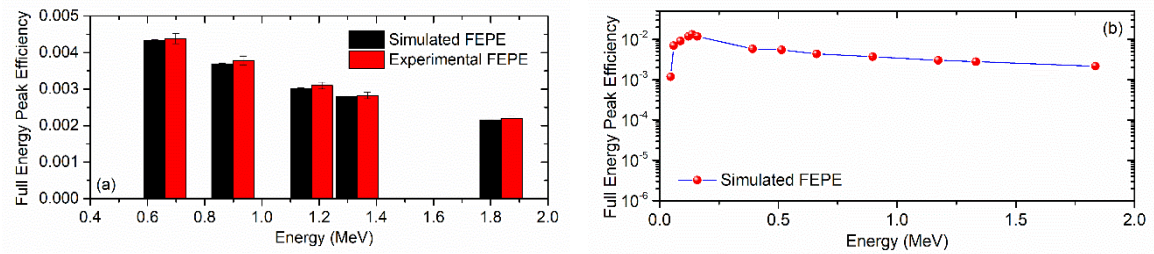

Figure 3. *Comparison between experimental and simulated FEPE (Figure 3a); Simulated FEPE values for a point source positioned at a distance of 10 cm (Figure 3b)*

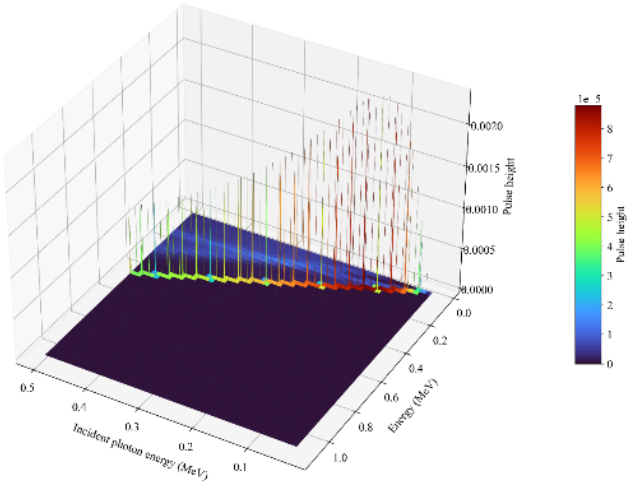

Figure 4. *Response function of the HPGe detector*

Gamma spectrum unfolding using equation (4) requires a detector response function matrix. While this matrix can, in principle, be built from experimental measurements with monoenergetic gamma sources of known activity, practical constraints make this approach infeasible. Gamma spectrometry systems often have thousands of energy channels, necessitating an impractically large number of sources to cover the entire energy range. To address this limitation, the response matrix for the HPGe detector was generated using the Monte Carlo simulation code MCNP6. Each response function was simulated with 4×10^8 particle histories to achieve a statistical uncertainty below 0.5%. The full simulation range spans 0.030147 MeV to 2.7984 MeV; however, as this study focuses on unfolding gamma peaks below 1.0 MeV, incident

photon energies were simulated from 0.030147 MeV to 1.0 MeV. Energy points were chosen at uniform intervals of 0.000171 MeV, matching the experimental spectrum's channel width. This synchronisation ensures strong compatibility between the simulated response matrix and measured data, improving the accuracy and reliability of the unfolding process (see Figure 4).

3.2. The result of the gamma spectrum unfolding using the UMG program

3.2.1. Validation of the unfolding model using the Monte Carlo simulation

In MCNP, the SDEF card provides flexibility in defining the radiation source, allowing the specification of particle type, source geometry, position, and energy distribution. The SI and SP card pairs define the gamma energy spectrum and associated weighting factors. In this study, a set of synthetic spectra was constructed using two gamma peaks: E_1 = 59.5 keV (^{241}Am) and E_2 = 661.7 keV (^{137}Cs), with varying weighting ratios to evaluate the accuracy of the unfolding model. Seventeen spectra, labelled AC1 to AC17, correspond to E_1: E_2 weight ratios ranging from 0.1: 0.9 (AC1 spectrum, where 10% of the emitted gamma rays have energy E_1 and 90% have energy E_2), 0.15: 0.85 (AC2), ... up to 0.9: 0.1 (AC17). In addition, the monoenergetic spectra at energies E_1 = 59.5 keV (^{241}Am) and E_2 = 661.7 keV (^{137}Cs) were also simulated to obtain the peak area corresponding to each monoenergetic energy peak ($N_0^{59.5\ keV}$ and $N_0^{661.7\ keV}$). Each spectrum was normalized to a total count rate of 37,000 counts/s (activity of 1μCi). Peak areas were determined using the Region of Interest (ROI) method in the Colegram software. The ω weighting factor at each energy peak is calculated as the ratio between the peak count in the composite spectrum AC (N_{AC}) and the peak count in the corresponding monoenergetic reference spectrum (N_0): $\omega = N_{AC} / N_0$. The relative uncertainty was assessed using the uncertainty propagation method. The results, shown in Figure 5a, indicate that the calculated weight for the 661.7 keV peak (^{137}Cs) deviates by a maximum of 1.3% from the initially assigned value, demonstrating the reliability of the simulated spectra.

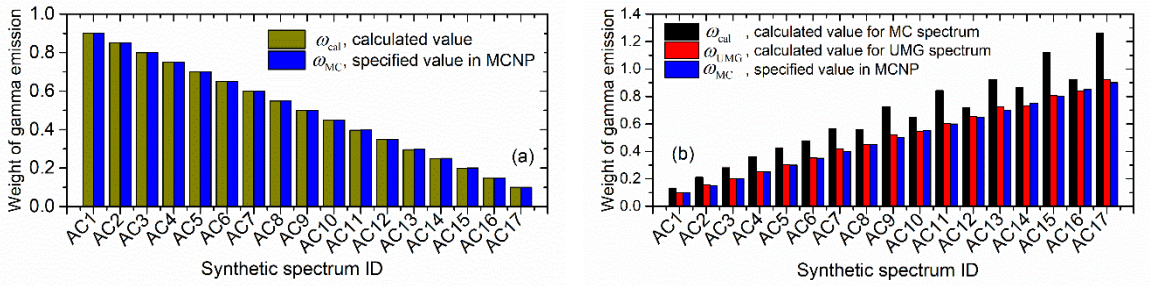

Figure 5. *Gamma emission weights were calculated from the simulated spectra: the 661.7 keV peak (^{137}Cs) (Figure 5a) and the 59.5 keV peak (^{241}Am) (Figure 5b)*

In contrast, Figure 5b shows that the evaluated weights for the 59.5 keV peak (^{241}Am) from the simulated spectra exhibit relatively large deviations, ranging from 8.7% to 45.0%. However, after applying the spectrum unfolding model, the error decreased significantly, remaining between 0.3% and 4.3%. The scientific novelty of this study lies in the development and validation of a quantitative method for evaluating the accuracy of gamma spectrum

unfolding models. This method allows for precise control over energy weights in multi-peak simulated spectra, a key factor in verifying the model's capability in deconvolving the true spectrum from overlapped measured signals (see Figure 6a).

3.2.2. Application of gamma spectrum unfolding to improve the measurement of radioisotope activity

The activity of each radionuclide at energy peak *E* (keV) was calculated using the following equation (Tran et al., 2018):

$$A_i = \frac{N_p(E_i)}{\varepsilon_p(E_i) \times I_\gamma(E_i) \times t} \times \Pi C_i \quad (8)$$

Where $N_p(E_i)$, $\varepsilon_p(E_i)$, $I_\gamma(E_i)$, and t represent the peak area, full-energy peak efficiency (FEPE), gamma emission intensity, and live time, respectively. The term ΠC_i includes correction factors such as coincidence summing, self-absorption, and decay-time corrections. However, for standard measurements at 10 cm, these corrections are negligible. The activity uncertainty includes contributions from uncertainties in peak counts, FEPE value, and gamma emission intensity, and is calculated using standard uncertainty propagation methods (GUM, 2008).

The unfolded gamma spectrum at the 59.5 keV peak (^{241}Am) is shown in Figure 6b. The net counts at the 59.5 keV energy peak of the unfolded spectrum increased by approximately 1.98 times compared to the original experimental spectrum, demonstrating the effectiveness of the proposed method. The activity of ^{241}Am at the 59.5 keV peak, when determined using the unfolded spectrum, shows a relative deviation of less than 3.3% compared to the reference activity value (Figure 7).

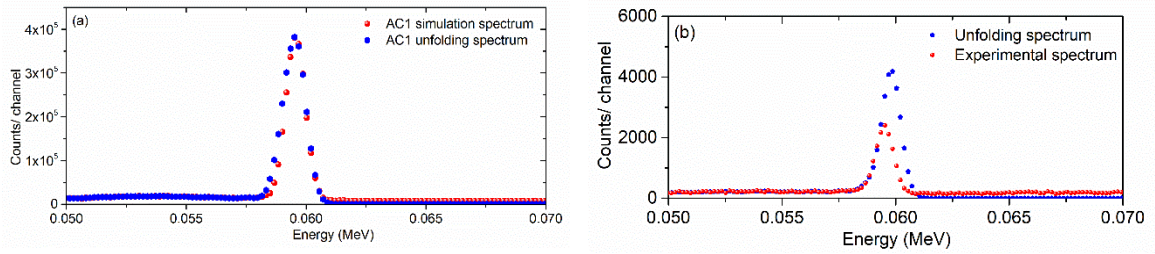

Figure 6. *The unfolded gamma spectrum at 59.5 keV from the simulated AC1 spectrum (Figure 6a) and the experimental spectrum (Figure 6b)*

Table 1 compares the reference activity, the activity from the experimental spectrum, and that obtained after applying the unfolding method. The measurement uncertainty was calculated using the error propagation method, in which the uncertainties arising from the simulated values were negligible and could be ignored. Figure 7 shows the relative deviations from the reference. Deviations using the experimental spectrum range from 13.1% to 47.8%, while those after dropped significantly to between 1.5% and 4.3%. This demonstrates that the unfolding method significantly improves activity determination accuracy and holds strong potential for practical gamma spectral analysis, especially below 500 keV.

Table 1. *Activity of radionuclides determined using the spectral unfolding method*

Radio-nuclides	Energy (keV)	Intensity (%)	Activity (Bq)		
			Certified value	Measured value	Unfolding value
^{210}Pb	46.5	4.252 ± 0.04	1114 ± 46	838.8 ± 38.2	1142 ± 51
^{241}Am	59.5	35.92 ± 0.17	111.6 ± 3.4	58.3 ± 1.9	115.3 ± 3.6
^{109}Cd	88.0	3.66 ± 0.05	668.9 ± 20.7	496.4 ± 17.5	693.1 ± 24.2
^{57}Co	122.1	85.49 ± 0.14	16.1 ± 0.5	12.5 ± 0.4	16.8 ± 0.6
^{57}Co	136.5	10.71 ± 0.15	16.1 ± 0.5	12.2 ± 0.5	16.5 ± 0.7
^{123m}Te	159.0	83.99 ± 0.08	7.4 ± 0.2	5.7 ± 0.2	7.2 ± 0.2
^{113}Sn	391.7	64.97 ± 0.17	25.6 ± 0.8	18.6 ± 0.6	24.5 ± 0.8
^{85}Sr	514.0	98.50 ± 0.40	6.3 ± 0.2	5.5 ± 0.2	6.4 ± 0.2

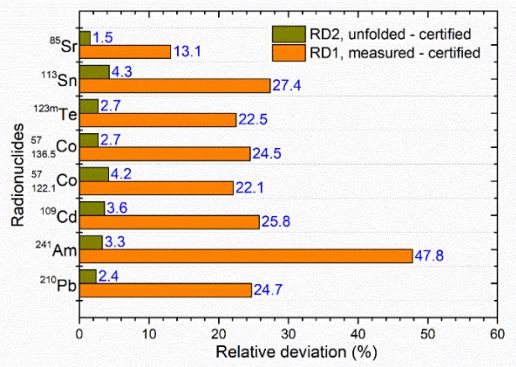

Figure 7. *Relative deviation between the activity determined by the experimental gamma spectrum (RD1, orange) and from the unfolded gamma spectrum (RD2, olive green), compared to the reference activity value*

4. Conclusions

This study proposes a method for determining radionuclide activity using gamma spectrum unfolding with the GRAVEL algorithm in the UMG program, enhanced by Monte Carlo simulations to improve accuracy. The key steps include: *(i)* validating the HPGe detector geometry using MCNP6 by comparing simulated and experimental spectra; *(ii)* constructing the detector response function for energies below 1 MeV; *(iii)* simulating gamma spectra with two peaks (^{241}Am and ^{137}Cs) at varying emission ratios, then verifying the model by comparing unfolded and input ratios; *(iv)* applying the method to spectra from a multi-radionuclide source placed 10 cm from the detector; and *(v)* calculating radionuclide activities from the unfolded spectra. The results show that relative deviations from reference activities remain below 4.3%, demonstrating superior accuracy over direct analysis of experimental spectra. This method is especially effective for low-energy gamma emitters, streamlining activity evaluation once the response matrix is established. However, it requires a well-constructed response matrix and prior knowledge of radionuclides to ensure correct peak unfolding. Despite these challenges, the unfolding method remains a valuable solution for gamma spectral analysis in low-activity environmental sample measurements, nuclear waste monitoring, and nuclear medicine applications.

❖ ***Conflict of Interest:*** *Authors have no conflict of interest to declare.*

❖ ***Acknowledgments:*** *This research is funded by Ho Chi Minh City University of Education Foundation for Science and Technology under grant number CS.2023.19.58.*

REFERENCES

Baré, J., & Tondeur, F. (2011). Gamma spectrum unfolding for a NaI monitor of radioactivity in aquatic systems: Experimental evaluations of the minimal detectable activity. *Applied Radiation and Isotopes*, *69*(8), 1121-1124. https://doi.org/https://doi.org/10.1016/j.apradiso.2010.11.024

Byon, J., Park, S., Kim, Y., & Ahn, S. (2022). External exposure specific analysis for radiation worker in reuse of containment building for Kori Unit 1. *Nuclear Engineering and Technology*, *54*(5), 1781-1788. https://doi.org/https://doi.org/10.1016/j.net.2021.11.015

Chen, Y., Chen, X., Lei, J., An, L., Zhang, X., Shao, J., Zheng, P., & Wang, X. (2014). Unfolding the fast neutron spectra of a BC501A liquid scintillation detector using GRAVEL method. *Science China Physics, Mechanics & Astronomy*, *57*(10), 1885-1890. https://doi.org/10.1007/s11433-014-5553-7

Dombrowski, H. (2024). Comparison of the conversion method with gravel unfolding to obtain dose values from photon spectra. *Radiation Protection Dosimetry*, *200*(1), 32-59. https://doi.org/10.1093/rpd/ncad251

Dryak, P., Kovar, P., & Gudelis, A. (2010). Total efficiency of GE detectors—dead layer signal effect. *Applied Radiation and Isotopes*, *68*(7), 1451-1453. https://doi.org/https://doi.org/10.1016/j.apradiso.2009.11.026

Eckert & Ziegler. *Mixed nuclide solutions*. https://www.ezag.com/products/isotope-products/isotrak-calibration-sources/standardized-solutions/calibrated-solutions/mixed-nuclide-solutions/

GUM. (2008). Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement. *JCGM 100:2008*. http://www.bipm.org. http://www.bipm.org

He, J.-F., Yang, Y.-Z., Qu, J.-H., Wu, Q.-F., Xiao, H.-L., & Yu, C.-C. (2016). An inversion decomposition method for better energy resolution of NaI(Tl) scintillation detectors based on a Gaussian response matrix. *Nuclear Science and Techniques*, *27*(3), 58. https://doi.org/10.1007/s41365-016-0062-1

Le, T. N. T., Huynh, D. C., & Tran, T. T. (2019). Efficiency calibration for HPGe detector by Monte Carlo efficiency transfer method. *Science & Technology Development Journal - Natural Sciences*, *3*(1), 9-17.

Li, F., Gu, Z., Ge, L., Li, H., Tang, X., Lang, X., & Hu, B. (2019a). Review of recent gamma spectrum unfolding algorithms and their application. *Results in Physics*, *13*, 102211. https://doi.org/https://doi.org/10.1016/j.rinp.2019.102211

Li, F., Lang, X., Chen, Y., Ge, L., Feng, L., & Li, S. (2019b). Evaluation on least square method applied to gamma spectrum de-noising. *Journal of Physics: Conference Series*, *1423*(1), 012002. https://doi.org/10.1088/1742-6596/1423/1/012002

Li, X., Wang, Z., Lv, H., Wei, S., Liu, J., Wang, Y., Fan, X., Zhou, R., & Yang, C. (2020). Measurement of the energy of fast neutrons in the presence of gamma rays using a NaI(Tl) and a plastic scintillator. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, *976*, 164257. https://doi.org/10.1016/j.nima.2020.164257

Liu, B., Yang, H., Lv, H., Jing, F., Gao, X., & Yan, M. (2020). A deconvolution method for scintillator gamma-ray spectrum analysis based on convex optimization. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, *957*, 163399. https://doi.org/https://doi.org/10.1016/j.nima.2020.163399

Maestro-32. (2003). *Software user's manual vesion 6*.

Ménesguen, Y., & Lépy, M. C. (2021). COLEGRAM, a flexible user-friendly software for processing of ionizing radiation spectra. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment, 1003*, 165341. https://doi.org/https://doi.org/10.1016/j.nima.2021.165341

Pelowitz, P. (2013). *MCNP6^{TM} User's manual, Version 1.0*. Los Alamos National Laboratory report LA-CP-13-00634.

PTB. (2004). *UMG 3.3, Analysis of data measured with spectrometers using unfolding techniques*. https://www.oecd-nea.org/tools/abstract/detail/nea-1665/

Tran, T. T., Le, Q. V., Phan, L. H., Huynh, D. C., Vo, H. N., & Chau, V. T. (2018). Validation of an advanced analytical procedure applied to the measurement of environmental radioactivity. *Journal of Environmental Radioactivity, 184-185*, 109-113. https://doi.org/10.1016/j.jenvrad.2017.12.020

Truong, T. H. L., Cao, P. L. H., Nguyen, D. P., Dang, T. T. M., & Ngo, H. Q. (2016). Study on analyzing overlaped peaks using the Gold algorithm of unfolding. *VNUHCM Journal of Science and Technology Development, 19*(2), 71-79. https://doi.org/10.32508/stdj.v19i2.791

XÁC ĐỊNH HOẠT ĐỘ ĐỒNG VỊ PHÓNG XẠ DỰA TRÊN PHỔ GAMMA ĐÃ GIẢI CUỘN BẰNG PHƯƠNG PHÁP GRAVEL VÀ MA TRẬN ĐÁP ỨNG TỪ MÔ PHỎNG MCNP

Lê Quang Vương[1,2*], Nguyễn Huỳnh Khánh Duyên[1], Nguyễn Huỳnh Duy Khang[1], Hoàng Đức Tâm[1], Phan Long Hồ[2,3], Trần Thiện Thanh[2], Châu Văn Tạo[2]

[1]Trường Đại học Sư phạm Thành phố Hồ Chí Minh, Việt Nam

[2]Trường Đại học Khoa học Tự nhiên, Đại học Quốc gia Thành phố Hồ Chí Minh, Việt Nam

[3]Viện Y tế Công cộng Thành phố Hồ Chí Minh, Việt Nam

[*]Tác giả liên hệ: Lê Quang Vương – Email: vuonglq@hcmue.edu.vn

Ngày nhận bài: 07-6-2025; Ngày nhận bài sửa: 31-10-2025; Ngày duyệt đăng: 18-11-2025

TÓM TẮT

Việc xác định chính xác hoạt độ phóng xạ đóng vai trò quan trọng để đảm bảo an toàn bức xạ và giám sát môi trường. Nghiên cứu này đề xuất một phương pháp kết hợp giữa giải cuộn phổ gamma sử dụng thuật toán GRAVEL và mô phỏng đáp ứng đầu dò bằng Monte Carlo nhằm nâng cao độ chính xác của kết quả phân tích hoạt độ. Một ma trận đáp ứng chi tiết được xây dựng bằng mã MCNP6 cho đầu dò HPGe loại p, với dải năng lượng photon lên đến 1 MeV, trong đó mỗi năng lượng photon mô phỏng tương ứng với các kênh năng lượng trong phổ gamma đo thực nghiệm. Phương pháp đề xuất cho thấy hiệu quả rõ rệt trong việc giải cuộn phổ, đặc biệt ở vùng năng lượng thấp như 59,5 keV, nơi thường bị ảnh hưởng mạnh bởi nhiễu nền và giới hạn của hệ đo. Kết quả kiểm chứng với nguồn điểm chứa nhiều đồng vị phóng xạ có hoạt độ chuẩn đã biết cho thấy phương pháp này giúp cải thiện đáng kể khả năng tách đỉnh và xác định chính xác hoạt độ phóng xạ. Độ lệch tương đối lớn nhất giữa giá trị đo và giá trị tham chiếu nhỏ hơn 4,3%, chứng minh tính hiệu quả và độ tin cậy của phương pháp, đặc biệt đối với các đồng vị phát gamma năng lượng thấp và cường độ yếu.

Từ khóa: hiệu suất ghi đỉnh; phổ kế gamma; mô phỏng MCNP; hoạt độ đồng vị phóng xạ; giải cuộn phổ

SUPPLEMENTARY INFORMATION

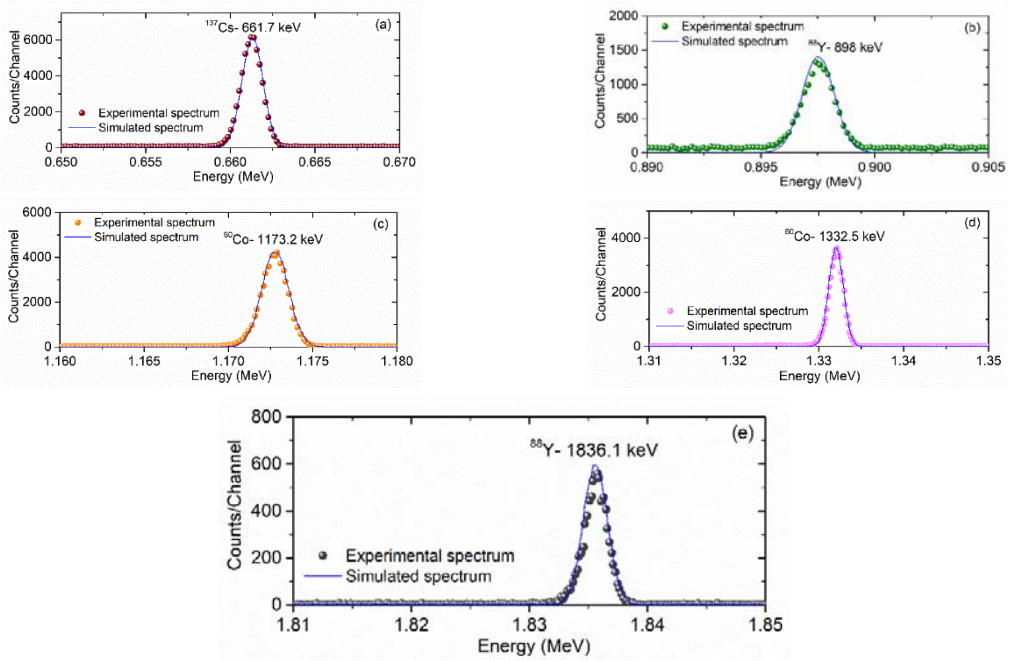

Figure S1. *MCNP6 simulation results of the gamma spectrum at energies 661.7, 898, 1173.2, 1332, and 1836.1 keV*

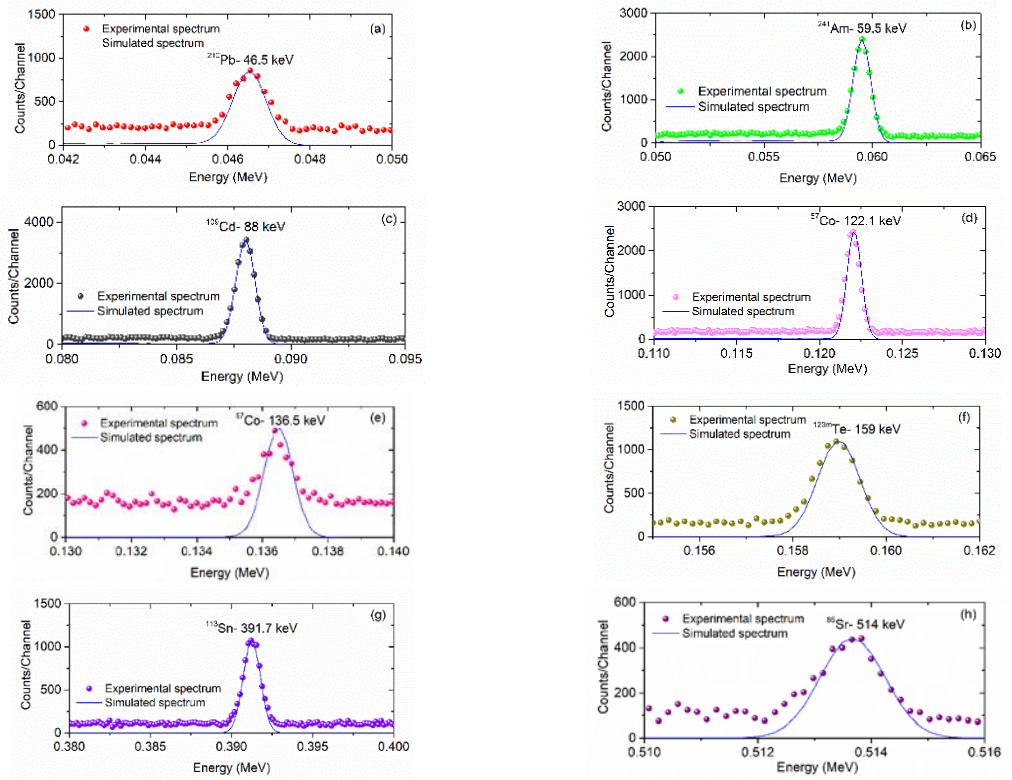

Figure S2. *MCNP6 simulation results of the gamma spectrum at energies 46.5, 59.5, 88.0, 122.1, 136.5, 159.0, 391.7, and 514.0 keV*