TẠP CHÍ KHOA HỌC
TRƯỜNG ĐẠI HỌC SƯ PHẠM TP HỒ CHÍ MINH
Tập 23, Số 9 (2026): 2034-2045
Website: https://journal.hcmue.edu.vn
ISSN: 2734-9918

HO CHI MINH CITY UNIVERSITY OF EDUCATION
JOURNAL OF SCIENCE
Vol. 23, No. 9 (2026): 2034-2045
https://doi.org/10.54607/hcmue.js.23.9.5481(2026)

Research Article

STUDENTS' MATHEMATICAL PROBLEM-SOLVING COMPETENCY IN A STAIRCASE-SLOPE MEASUREMENT TASK: A QUALITATIVE STUDY IN GRADE 9 TRIGONOMETRY

Tang Minh Dung*[*], *Le Nguyen Hoang Oanh
Ho Chi Minh City University of Education, Vietnam
[*]*Corresponding author: Tang Minh Dung – Email: dungtm@hcmue.edu.vn*
Received: January 4, 2026; Revised: June 22, 2026; Accepted: September 8, 2026

ABSTRACT

This qualitative study examines how ninth-grade students demonstrate mathematical problem-solving competency during a staircase-slope measurement task within the topic of trigonometric ratios of acute angles. Rather than testing whether the task improves competency, the study describes students' observable problem-solving processes in a contextual measurement scenario. In Ho Chi Minh City, 39 ninth-grade students worked in seven groups to identify criteria for a safe staircase, measure school staircases, calculate slope angles using trigonometric ratios (tangent, sine, and cosine), and present and justify their solutions. Data sources included observation notes, group worksheets and diagrams, photographs, and video recordings. Data were analyzed qualitatively using four consistent indicators of mathematical problem-solving competency. The results showed that students could formulate the staircase-slope situation as a mathematical problem, select feasible measurement plans, use trigonometric ratios appropriately, and compare their results with the safety standards. The study provides a qualitative description of how this contextual measurement task reveals students' mathematical problem-solving competency in Grade 9 trigonometry.

Keywords: Mathematical Problem-Solving Competency; Trigonometric Ratios; Slope

1. Introduction

1.1. Background and Research Problem

In the 21st century, education worldwide has been shifting from content-based approaches toward models emphasizing adaptive skills and transversal competencies. In Vietnam, the 2018 General Education Program reflects this orientation through competency-based education and requires mathematics education to connect with real-world situations (Ministry of Education and Training, 2018a). Trigonometric ratios of acute angles constitute an important topic in secondary school mathematics, especially in Grade 9. Students

Cite this article as: Tang, M. D., & Le, N. H. O. (2026). Students' Mathematical problem-solving competency in a staircase-slope measurement task: A qualitative study in Grade 9 trigonometry. *Ho Chi Minh City University of Education Journal of Science, 23*(9), 2034-2045. https://doi.org/10.54607/hcmue.js.23.9.5481(2026)

progress from known geometric relationships to more abstract relationships involving ratios, angles, and similar triangles. However, this progression is not always straightforward. When trigonometry teaching relies on lecture-based instruction, memorized formulas, and board drawings, students may view it as fixed rules rather than a way to describe spatial relationships. This can make trigonometry a disconnected, meaningless procedure, as noted by Arhin and Hokor (2021). This difficulty is also associated with the epistemological barrier to learning trigonometry (Schneider, 2014). Students commonly have difficulty reconceptualizing ratios as structural objects that express relationships in similar triangles rather than as calculation procedures (Sfard, 1991). As a result, students might not realize that trigonometric ratios vary depending on the angle measure and that they can be used to solve real-world spatial problems. Outdoor measurement tasks provide meaningful contexts for connecting trigonometric ratios with real-world spatial relationships. From an embodied cognition perspective, this connection is significant because students' mathematical ideas may be reinforced through bodily movement, spatial reasoning, and manipulation of physical objects in their surroundings (Abrahamson & Bakker, 2016; Lakoff & Núñez, 2000).

Although research on applied learning, contextual learning, mathematical modeling, and real-world applications in mathematics education has been conducted in Vietnam, further qualitative research is needed to describe how ninth-grade students demonstrate mathematical problem-solving competency during short, focused outdoor measurement tasks. Specifically, little empirical research has examined students' processes of identifying problems, selecting and applying trigonometric strategies, and checking results during staircase-slope measurement tasks. This study explores and describes manifestations of students' mathematical problem-solving competency during the task. The guiding research question is as follows: How do ninth-grade students demonstrate mathematical problem-solving competency when engaging in a contextual staircase-slope measurement task?

1.2. Literature Review and Theoretical Frameworks

1.2.1. Applied and Contextual Learning in Mathematics Education

This study draws on applied and contextual learning perspectives, which hold that students learn mathematics by engaging with meaningful real-world situations. This paradigm shift is supported by the constructivist epistemologies of Piaget (1971) and Vygotsky (1978). From a constructivist perspective, students actively construct knowledge through interaction with their environment, including tasks, tools, and peers.

Applied and contextual approaches focus on tasks in which students can mobilize and assess their existing knowledge in meaningful contexts rather than develop new concepts (Blum & Leiß, 2007; Lesh & Zawojewski, 2007). The cognitive challenge lies not in conceptual novelty but in selecting relevant knowledge, adjusting it to contextual constraints, and checking the reasonableness of the outcomes. This aligns with Schoenfeld's (2016)

definition of mathematical problem solving, which consists of understanding the situation, choosing and monitoring problem-solving strategies, and evaluating results.

1.2.2. Mathematical Problem-Solving Competency

Problem solving has long been studied in mathematics education, and the concept has continued to evolve over time. Pólya (1957) proposed four steps: understand the problem, make a plan, carry out the plan, and review. Schoenfeld (2016) subsequently argued that effective problem solvers need to understand the situation, select and monitor strategies, use mathematical resources appropriately, and evaluate the reasonableness of results. He also clearly distinguished between exercises, for which the solver already had a solution procedure, and problems, for which the solver had to develop a new approach.

Subsequent research has shifted the focus from problem solving as a cognitive process to a measurable competency. For example, the OECD's Program for International Student Assessment (PISA) views mathematical literacy through two aspects: mathematical reasoning and problem solving. In this context, mathematical literacy plays a crucial role in solving real-world problems (OECD, 2023). This understanding of problem solving is closely connected to mathematical modeling. In modeling tasks, students usually begin with a situation outside mathematics, represent it in mathematical terms, work with the mathematical model, and then interpret the result in relation to the original context (Blum & Leiß, 2007). Studies on modeling have shown that tasks involving measurement and estimation can create opportunities for higher-order thinking because students must decide what information is relevant, make assumptions, and check whether their results are reasonable (English & Watson, 2016; Lesh & Zawojewski, 2007).

According to the Ministry of Education and Training (2018a), problem-solving and creativity competency is identified as one of the core general competencies. Specifically, mathematical problem-solving competency is one of the five mathematics-specific competencies to be developed across grade levels (Ministry of Education and Training, 2018b). In this study, students' mathematical problem-solving competency is examined through four indicators that are derived from the 2018 General Education Program: (1) recognize and identify problems that need to be solved using mathematics; (2) select and propose methods and solutions to solve the problem; (3) use appropriate mathematical knowledge and skills (including tools and algorithms) to solve the given problem; (4) evaluate the proposed solution and generalize it to similar problems. These four indicators correspond, respectively, to the phases of problem recognition, strategy selection, mathematical execution, and contextual evaluation that recur across the work of Schoenfeld (2016), Blum and Leiß (2007), and Lesh and Zawojewski (2007).

The staircase-slope measurement task is not a nonroutine problem in Schoenfeld's (2016) sense because the students have already learned the definitions and applications of trigonometric ratios. However, the primary cognitive challenge involves mathematizing a

physical object rather than performing calculations. Students must independently recognize that a staircase can be represented as a right triangle, determine which measurable dimensions correspond to its sides, and select a trigonometric ratio appropriate to their measurements. They must also interpret their calculated angle in relation to an external safety standard, making judgments that are not usually required in a classroom context.

2. Participants and Research Methodology

2.1. Participants

The study involved 39 ninth-grade students in Ho Chi Minh City, Vietnam. The task took place during the first semester, after the students had completed classroom lessons on the trigonometric ratios of acute angles. Students worked in seven groups of five to seven during the outdoor activity. The teacher did not provide a fixed procedure for solving the task. In addition to supporting the groups, the teacher organized the activity, ensured safety during outdoor measurement, facilitated discussion when needed, and asked open-ended questions to help students explain their thinking.

2.2. Scenario

The task used a familiar school structure—a staircase—as a context for applying trigonometry. In this study, the task is understood as an applied and contextual learning activity in which students draw on prior trigonometric knowledge to address a practical measurement problem in their school environment.

The activity was organized into three phases. In Phase 1, a 45-minute period, students were divided into groups to discuss what makes a staircase compliant with standards or safe. They searched online for relevant information, identified two possible criteria based on their findings, and presented them to the class. In the whole-class discussion, students established a shared basis for discussing staircase slope. At the end of this phase, the teacher summarized the students' ideas and chose slope as the primary criterion for the next stage of the task. Phase 2 involved 30 minutes of outdoor activity. The groups used a staircase in the school, recorded its location, and measured the required dimensions, such as the height and width of the steps and the length of the inclined side. They then used an appropriate trigonometric ratio to determine the slope angle. The groups filled out a worksheet in which they were required to record the staircase's location and measurements, the steps taken in the calculation, and their assessment of whether the staircase was safe or unsafe. In Phase 3, which lasted 45 minutes, the groups presented their methods and results to the class. Group representatives explained how they measured the staircase, which trigonometric ratio they used, and what conclusion they reached. Other students asked questions, commented on the methods, and compared different approaches.

2.3. Data Collection

Students' mathematical problem-solving competency was explored qualitatively using multiple data sources collected during the three phases of the activity to support

triangulation. The primary data sources were observation notes, group worksheets and diagrams, photographs of the measurement process, and video recordings of presentations.

2.4. Data Analysis

Data on problem-solving competency were analyzed qualitatively using content analysis, with reference to the four core indicators outlined in the 2018 General Education Program in Mathematics. Specifically, students' statements from video transcripts and their calculation steps on worksheets were manually coded and categorized according to four indicators:

(1) Recognize and identify problems that need to be solved using mathematics: determining whether the staircase slope was within the safety standard range $(20° - 45°)$.

(2) Select and propose methods and solutions to solve the problem: proposing a suitable measurement plan (height, width, hypotenuse), calculations, and the tools needed.

(3) Utilize appropriate mathematical knowledge and skills to solve the given problem: using trigonometric ratios (sine, cosine, tangent) to determine the angle of the slope as accurately as possible.

(4) Evaluate the proposed solution and generalize it to similar problems: analyzing the effectiveness of various strategies to solve problems, reflecting on their own strategy, and defending their arguments when discussing alternative solutions. In this study, this indicator was observed only in the evaluation and justification aspects; the generalization aspect was outside the scope of the task.

The indicators can be systematically triangulated across data sources (worksheets, photographs, and video recordings) to describe how students transform a real-world problem into a mathematical model, develop solutions, and present them in an outdoor learning environment.

3. Findings and Discussion

3.1. Findings

The results of the lesson scenario, namely the measurements and calculations of staircase slopes at the school, were analyzed qualitatively with respect to the four indicators of mathematical problem-solving competency used in this study.

3.1.1. Recognize and Identify Problems That Need to Be Solved Using Mathematics

Students collaborated in groups to determine the factors indicating whether a staircase meets the expected design requirements. Of the 14 ideas proposed by the seven groups, approximately half concerned two important mathematical factors: the inclination angle (stair slope) and the stair riser height and tread width. Broad guidelines suggest that the inclination angle should be between 20 degrees and 45 degrees with respect to the horizontal plane (Group 1). Stricter comfort criteria state that common staircases typically have an inclination angle between $18 - 33$ degrees, and the riser height (H) should be $120mm - 170mm$, which is regarded as the most comfortable to use regularly (Group 3).

An absolute safety limit also applies: the inclination of staircases should not exceed 65 degrees (Group 4). The stair riser height in individual dimensions must be between 13 and 22.5 cm in general (Group 1), with the stair tread width ranging from 21.5 to 35.5 cm (Group 5), with detailed codes stating that the tread must be 280 mm wide and the height of the riser must not exceed 180 mm (Group 7). More importantly, the combined dimensions are governed by the dimensional rule (2H + B), according to which the sum of twice the riser height (H) and the tread width (B) should be no less than 550 mm and no more than 700 mm (Group 2). At the end of the discussion, the teacher instructed each group to rely on the first standard, according to which the inclination angle of the staircase should be 20°- 45° with the horizontal plane. This standard was selected as the primary mathematical reference point for subsequent stages of problem solving.

3.1.2. Select and Propose Methods and Solutions to Solve the Problem

During this step, students selected appropriate trigonometric strategies and applied their knowledge to compute the slope angle using the obtained data. Three main solution strategies emerged, illustrating students' different choices of measurement and calculation methods (Table 1).

Table 1. *Student Groups' Measured Quantities and Calculations*

Strategy	Group	Measured Quantities	Calculation
Tangent	1	Height $h = 13$ Width w= 30	$\tan\alpha = \frac{h}{w} = \frac{13}{30}$ $\alpha \approx 23°25'$
	3	Height $h = 14.5$ Width w= 29	$\tan\alpha = \frac{h}{w} = \frac{14.5}{29}$ $\alpha \approx 26°33'$
Sine	6	Height $h = 14.5$ Width $w = 31$	Hypotenuse $c = \sqrt{h^2 + w^2} \approx 34.2$ $\sin\alpha = \frac{h}{c} = \frac{14.5}{34.2}$ $\alpha \approx 25°5'$
	4	Height $h = 14.5$ cm Hypotenuse $c = 32$ cm	$\sin\alpha = \frac{h}{c} = \frac{14.5}{32} \approx 0.45$ $\alpha \approx 26°57'$
Cosine	2	Hypotenuse $c = 32.7$ cm Width $w = 30$ cm	$\cos\alpha = \frac{w}{c} = \frac{30}{32.7}$ $\alpha \approx 23°27'$
	7	Width $w = 31$ cm Hypotenuse $c = 35$ cm	$\cos\alpha = \frac{w}{c} = \frac{31}{35}$ $\alpha \approx 27.7°$
	5	Hypotenuse $c = 35.5$ Width w= 30.8	$\cos\alpha = \frac{w}{c} = \frac{30.8}{35.5}$ $\alpha \approx 29.82°$

3.1.3. Utilize Appropriate Mathematical Knowledge and Skills to Solve the Given Problem

To calculate the staircase's slope angle, all groups used trigonometric ratios (tangent, sine, or cosine) with measurements collected directly during the activity. Most groups used

the measured values of height (h), width (w), or hypotenuse (c) to compute the ratio. The calculations showed that the groups were generally able to apply trigonometric formulas to determine the slope angle, and the results fell within the selected safety-standard range of $(20° - 45°)$.

3.1.4. Evaluate the Proposed Solution and Generalize It to Similar Problems

To explore students' problem-solving processes further, the teacher invited three representative groups (Groups 1, 2, and 6) to participate in the discussion. These groups were purposively selected because each used a different trigonometric ratio (tangent, cosine, or sine) and demonstrated distinct measurement and data analysis methods, illustrating different ways of applying prior mathematical knowledge to staircase slope.

Group 1 chose the stairway on the second floor of block A. The students measured the width and height of one step. They then modeled the step as a right triangle and used the tangent ratio to compute the angle of inclination:

> First, from the edge of the upper step, we drew a perpendicular line down to the edge of the lower step and measured AB $=$ 13 cm the height of the step. Then, from that perpendicular point, we drew a horizontal line to the edge of the lower step and measured $AC = 30\ cm$, which is the width of the step. Finally, we applied the trigonometric ratio $\tan\alpha = \frac{\mathrm{AB}}{AC}$ to calculate the staircase's slope angle.
>
> Applying the trigonometric ratio implies:
>
> $\tan\alpha = \frac{AB}{AC} = \frac{13}{30}$
>
> $\alpha = 23°25'$.
>
> Therefore, the staircase meets the standard requirement.

Other students noted that Group 1's method was "clear and easy to follow" because it precisely showed how the right triangle was constructed from the staircase measurements. They also noted that using the tangent ratio was the most appropriate choice given the available data (height and width).

Group 2 selected the staircase on the third floor of Building D. The students used measurements of the width and hypotenuse of the step. They applied the cosine ratio to find the inclination angle of the staircase:

> We measured the distance from the edge of the step to the tip of the next step, which gave the hypotenuse BC = 32.7 cm. Then, we measured the horizontal distance of the step, AC = 30 cm. Using these values, we applied the trigonometric ratio
>
> $\cos C = \frac{AC}{BC} = \frac{30}{32.7} \approx 0.92$
>
> which gives $C \approx 23°27'$. Therefore, the staircase has a relatively steep slope.

Several students appreciated Group 2's attempt to measure the inclined hypotenuse directly, noting that this approach offered a different perspective from Group 1's use of vertical and horizontal measurements. A fellow group member described direct measurement

of the inclined line as inventive but difficult to perform precisely because the tilted surface made accurate ruler placement challenging.

Group 6 investigated the staircase in the second-floor corridor. The students reported measurements of the height and width of one step and used trigonometric reasoning to determine the slope angle and compare it to the safety standard range ($20° - 45°$):

> The ruler was put on the edge of the upper step then down to the lower step in a vertical position to measure the height of the step AB=14.5 cm. Then, the ruler was laid horizontally on the edge of the nosing of the step to measure the width AC=31 cm.
>
> Considering the right triangle ABC, we have:
>
> $BC^2 = AB^2 + AC^2 = 14.5^2 + 31^2 \Rightarrow BC \approx 34.2$
>
> Applying the trigonometric ratio:
>
> $\sin C = \frac{AB}{BC} = \frac{14.5}{34.2} \approx 0.424$
>
> Thus, $C \approx 25°5'$.
>
> Since $20° < 25°5' < 45°$, the slope of the staircase meets the safety standard.

Students commended Group 6 for demonstrating a more comprehensive process by using the Pythagorean theorem to compute the hypotenuse before applying trigonometry. However, a peer noted that indirect calculation could introduce rounding errors and proposed comparing the computed hypotenuse with the measured inclined length to verify the answer.

Overall, the classroom discussion showed that students did not merely report numerical answers. They also explained why a particular trigonometric ratio was appropriate, compared the calculated angle with the safety standard, and commented on possible sources of measurement error. These observations provide evidence of students' ability to analyze and justify their solutions within the staircase-slope context.

3.2. Discussion

This study examined how ninth-grade students demonstrated mathematical problem-solving competency while working on a staircase-slope measurement task. The findings show how students expressed different aspects of problem-solving through their measurements, calculations, explanations, and justifications.

The finding that students grounded their trigonometric reasoning in direct physical measurement of the staircase is consistent with the embodied-cognition perspective introduced earlier, according to which mathematical concepts are supported by spatial reasoning and interaction with environmental objects (Abrahamson & Bakker, 2016; Lakoff & Núñez, 2000). By measuring actual staircases, students could relate their spatial intuition to formal trigonometric concepts such as angle, height, width, and ratio. Out-of-classroom learning (OCL) was identified by Waite (2017) as making a cognitive link between concrete and abstract ideas. Students' calculation of angles using actual staircase measurements aligns with the work of Abakah and Antwi (2024), which shows that experiential learning can aid visualization in 3D trigonometry problems. The results also align with the work of Kissane

and Kemp (2009) and Nguyen et al. (2024) regarding the relationship between the trigonometric ratios and the familiar geometric relationships. The staircase-slope measurement task provided a concrete context that appeared to connect measurements, right-triangle representations, and trigonometric ratios.

The findings suggest that field-based measurement activities offer contexts for observing students' mathematical modeling processes. Students' skills in problem definition, choice of measurement strategy, use of trigonometric ratios, and justification of conclusions are consistent with the core problem-solving thinking described by Lesh and Zawojewski (2007). The results are also consistent with Ngcobo et al. (2019) and Maknun et al. (2019), who reported that students are more effective in applying trigonometric ratios in architectural and topographical measurements. In this study, the staircase safety scenario imposed contextual constraints that prompted students to explain their answers in a practical context. This reflects an important aspect of competency-based mathematics learning, in which students are expected to use mathematical knowledge to interpret and justify solutions in meaningful situations, which Tang and Vo (2024) emphasize in their article on spatial geometry.

Although some uniformity was observed in the early phases of the problem-solving process, differences emerged in the final phase. One limitation observed in the task was the limited depth of students' reflective justification. This is evident in the limited evidence for the fourth indicator, namely, the analysis and justification of the solution in context. The students in this study demonstrated good procedural skills in contrast to the successful conceptual gains documented in context-based trigonometry studies by Ngcobo et al. (2019). Nevertheless, the data primarily showed students' ability to calculate and compare their results against the selected safety standard, while their explanations of measurement errors and comparisons of strategies were still limited. The limited depth of students' justification may reflect the epistemological barrier in trigonometry (Schneider, 2014), which involves difficulty transitioning between ratios as calculation procedures and as structures. This may be the case even after teaching, as suggested by Dhungana et al. (2023). In the current task, students had the opportunity to use the trigonometric ratios to determine the angles they sought, but there were few opportunities for them to compare solution strategies or discuss why the 20°–45° range was selected as the safety standard.

The findings have implications for the theory and practice of mathematics education in general and the implementation of the 2018 General Education Program in Mathematics in particular. Theoretically, they contribute to discussions of applied and contextual learning by showing how outdoor measurement helps students connect abstract trigonometric ideas with a concrete problem in their school environment. The activity also provides a clear way to observe the four indicators of mathematical problem-solving competency required in the 2018 General Education Program in Mathematics. Students work directly with a real structure, measure, represent, and interpret quantitative relationships, and demonstrate

essential components of mathematical problem-solving competency. The activity also challenges the students to consider the value of trigonometry in determining safety and analyzing built environments, thus helping to make mathematical reasoning more meaningful and relevant. The task is simple, accessible, and adaptable to other inclines (e.g., ramps or roof slopes), making it a useful model for experience-oriented tasks that involve practical observation and mathematical analysis.

This study identifies limitations in assessing problem-solving competency. First, the intervention was too brief to develop the iterative thinking that is indicative of higher-order problem solving. Deeper aspects of competency, especially reflection and justification, may require sustained practice and scaffolding. Second, there were limitations in the pedagogical approaches used to implement the lesson. In Phase 1, the rationale for the selected safety standard was insufficiently explained, which limited students' deeper reasoning and rigorous problem formulation. During Phase 3, the lack of teacher-led synthesis restricted opportunities for reflection. Students had limited opportunities for guided reflection linking their data to mathematical understanding of slope, ratio, and angle.

Future research should use a longitudinal design to track manifestations and potential development of problem-solving competency through longer-term applied or contextual learning. Problem recognition may emerge within a single lesson, but several sessions are needed to gather stronger evidence of students' ability to analyze and justify contextual solutions. Future implementations should also include purposeful scaffolding during reflection. Finally, technology-based tools, such as digital angle sensors or dynamic geometry software, can minimize measurement errors and free up students to focus on analyzing and interpreting problems.

4. Conclusion

This study examined how ninth-grade students demonstrated mathematical problem-solving competency through a staircase-slope measurement activity aligned with the objectives of the 2018 General Education Program. Qualitative analysis identified observable aspects of this competency: Students could identify appropriate mathematical standards, select and use trigonometric problem-solving strategies (tangent, sine, and cosine), and critically evaluate their solutions against the selected safety standards. However, the study involved only one short-term task and therefore cannot establish whether students' problem-solving competency improved significantly.

❖ ***Conflict of Interest:*** *Authors have no conflict of interest to declare.*

❖ ***Acknowledgments:*** *This research is funded by Ho Chi Minh City University of Education Foundation for Science and Technology under grant number CS.2025.19.70.*

REFERENCES

Abakah, F., & Antwi, A. (2024). Using experiential learning in solving 3-dimensional problems in trigonometry. *Gulf Journal of Mathematics*, *16*(2), 364–379. https://doi.org/10.56947/gjom.v16i2.1817

Abrahamson, D., & Bakker, A. (2016). c. *Cognitive Research: Principles and Implications*, *1*(33), 1–13. https://doi.org/10.1186/s41235-016-0034-3

Arhin, J., & Hokor, E. K. (2021). Analysis of high school students' errors in solving trigonometry problems. *Journal of Mathematics and Science Teacher*, *1*(1), em003. https://doi.org/10.29333/mathsciteacher/11076

Blum, W., & Leiß, D. (2007). How do students and teachers deal with mathematical modelling problems?. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling (ICTMA12): Education, Engineering and Economics* (pp. 222–231). Horwood Publishing.

Dhungana, S., Pant, B. P., & Dahal, N. (2023). Students' experience in learning trigonometry in high school mathematics: A phenomenological study. *Mathematics Teaching Research Journal, 15*(4), 184–201. https://files.eric.ed.gov/fulltext/EJ1409234.pdf

English, L. D., & Watson, J. (2016). Modelling with authentic data in sixth grade. *ZDM Mathematics Education, 50*, 103–115. https://doi.org/10.1007/s11858-017-0896-y

Kissane, B., & Kemp, M. (2009). Teaching and learning trigonometry with technology. *The 14th Asian Technology Conference in Mathematics*, Beijing Normal University, Beijing, China. Mathematics and Technology, LLC. https://atcm.mathandtech.org/ep2009/papers_full/2812009_17288.pdf

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being.* Basic Books.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763–804). Information Age Publishing.

Maknun, C. L., Rosjanuardi, R., & Jupri, A. (2019). From ratios of right triangle to unit circle: an introduction to trigonometric functions. *Journal of Physics: Conference Series*, *1157*(2), 022124. https://doi.org/10.1088/1742-6596/1157/2/022124

Ministry of Education and Training. (2018a). *The General education program (issued together with Circular No. 32/2018/TT-BGDĐT dated December 26, 2018).*

Ministry of Education and Training. (2018b). *The General education program in Mathematics (issued together with Circular No. 32/2018/TT-BGDĐT dated December 26, 2018).*

Ngcobo, A. Z., Madonsela, S. P., & Brijlall, D. (2019). The teaching and learning of trigonometry. *Independent Journal of Teaching and Learning*, *14*(2), 72–91. https://hdl.handle.net/10520/EJC-1d6fb8bc84

Nguyen, T. D., Do, D. A., Pham, T. C. G., Phung, P. D., & Dang, B. K. L. (2024). Teaching trigonometry based on realistic mathematics education and game-based learning approach. *Journal of Instructional Mathematics*, *5*(2), 168–179. https://doi.org/10.37640/jim.v5i2.2173

OECD. (2023). *PISA 2022 Assessment and analytical framework*. OECD Publishing. https://doi.org/10.1787/dfe0bf9c-en

Piaget, J. (1971). *Psychology and epistemology: Towards a theory of knowledge*. Grossman Publishers.

Pólya, G. (1957). *How to solve it: A new aspect of mathematical method*. Princeton University Press.

Schneider, M. (2014). Epistemological obstacles in teaching mathematics. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 214–217). Springer. https://doi.org/10.1007/978-94-007-4978-8_57

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics (Reprint). *Journal of Education, 196*(2), 1–38.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics, 22*, 1–36. https://doi.org/10.1007/BF00302715

Tang, M. D., & Vo, N. H. (2024). Enhancing students' mathematical competence in learning spatial geometry in grade 11 through practical and experiential activities. *Dong Thap University Journal of Science*, *13*(7), 99–108. https://doi.org/10.52714/dthu.13.7.2024.1342

Vygotsky, L. (1978). *Mind in society: the development of higher psychological processes.* Harvard University Press. https://doi.org/10.2307/j.ctvjf9vz4

Waite, S. (2017). *Children learning outside the classroom: From birth to eleven*. Sage.

NĂNG LỰC GIẢI QUYẾT VẤN ĐỀ TOÁN HỌC CỦA HỌC SINH TRONG NHIỆM VỤ ĐO ĐỘ DỐC CẦU THANG: MỘT NGHIÊN CỨU ĐỊNH TÍNH TRONG LƯỢNG GIÁC Ở LỚP 9

Tăng Minh Dũng*, Lê Nguyễn Hoàng Oanh

Trường Đại học Sư phạm Thành phố Hồ Chí Minh, Việt Nam

**Tác giả liên hệ: Tăng Minh Dũng – Email: dungtm@hcmue.edu.vn*

Ngày nhận bài: 04-01-2026; Ngày nhận bài sửa: 22-6-2026; Ngày duyệt đăng: 08-9-2026

TÓM TẮT

Nghiên cứu định tính này khám phá cách học sinh lớp 9 biểu hiện năng lực giải quyết vấn đề toán học khi tham gia nhiệm vụ đo độ dốc cầu thang trong chủ đề tỉ số lượng giác của góc nhọn. Thay vì kiểm chứng việc hoạt động có làm nâng cao năng lực hay không, nghiên cứu mô tả các biểu hiện có thể quan sát được trong quá trình học sinh giải quyết một tình huống đo lường gắn với bối cảnh thực tế. Ba mươi chín học sinh lớp 9 tại Thành phố Hồ Chí Minh làm việc theo bảy nhóm để xác định các tiêu chí của cầu thang an toàn, đo đạc cầu thang trong trường, tính góc dốc bằng các tỉ số lượng giác (tang, sin, cos), sau đó trình bày và bảo vệ lời giải. Dữ liệu gồm ghi chú quan sát, phiếu học tập và sơ đồ nhóm, ảnh chụp và video phần trình bày. Dữ liệu được phân tích định tính theo bốn chỉ báo của năng lực giải quyết vấn đề toán học. Kết quả cho thấy học sinh có thể mô hình hóa tình huống độ dốc cầu thang thành một vấn đề toán học, lựa chọn chiến lược đo đạc khả thi, vận dụng phù hợp các tỉ số lượng giác, và đối chiếu kết quả với chuẩn an toàn đã cho. Nghiên cứu đóng góp một mô tả định tính về cách một nhiệm vụ đo lường theo bối cảnh có thể làm bộc lộ năng lực giải quyết vấn đề toán học của học sinh lớp 9 trong dạy học lượng giác.

Từ khoá: năng lực giải quyết vấn đề toán học; tỉ số lượng giác; độ dốc