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Research Article

SOME FIXED POINT THEOREMS IN CONE E_b-METRIC SPACES

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ABSTRACT

Fixed point theory plays a fundamental role in mathematics and has wide-ranging applications across scientific disciplines. In recent years, numerous studies have focused on extending classical fixed point results to generalized metric spaces, among which cone E_b-metric spaces have received particular attention. In this paper, we employ the method of Miculescu and Mihail (2017) to establish sufficient conditions under which sequences in cone E_b-metric spaces, with cones containing semi-interior points, are e-Cauchy. Our results extend the work of Miculescu and Mihail (2017) in cone E_b-metric spaces and further generalize the findings of Zahia et al. (2023) and Djedid et al. (2025) by relaxing the contraction coefficient from $0 \leq q < \frac{1}{s}$ to $0 \leq q < 1$. As a consequence, we derive the Hardy-Rogers type fixed point theorem, the Banach type fixed point theorem, and the Kannan type fixed point theorem in cone E_b-metric spaces. The results obtained not only generalize previously known theorems but also enrich the framework of fixed point theory in generalized metric spaces. Moreover, these contributions open a new research direction for applications in nonlinear analysis, differential equations, and optimization, thereby underscoring the relevance of cone E_b-metric spaces in addressing complex mathematical problems.

Keywords: Cone E_b-metric space; fixed point; semi-interior point

1. Introduction and preliminaries

The notion of a generalized metric space was introduced by Bogdan Rzepecki in 1980 by replacing the co-domain of real numbers in the notion of a metric space by a real Banach space (Rzepecki, 1980). Long-Guang and Xian (2007) reintroduced this notion under the name of a cone metric. Hussain and Shah (2011) extended the notion of the cone metric space and the b-metric spaces and named the cone b-metric spaces. Al-Rawashdeh et al. (2012) proposed the notion of E-metric space and characterized the cone metric spaces in a more general way by defining ordered normed spaces.

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In the majority of existing research, the obtained results have been formulated in the setting of the solid cone. Specifically, Du demonstrated that several fixed point results established in cone metric spaces with solid cones and real-valued contraction constants are consequences of the corresponding fixed point theorems in classical metric spaces (Du, 2010). Basile et al. (2017) introduced the notion of semi-interior points in a cone. An interesting observation is that every interior point is also a semi-interior point, whereas the converse does not hold. This distinction opens new directions of research involving cones with semi-interior points. Such investigations are more general than previous studies, which were restricted to cones with nonempty interiors. Moreover, this constitutes a genuine extension in the study of cones, as well as in the development of fixed point theorems and their applications to spaces endowed with cones possessing semi-interior points.

Djedid et al. (2023) introduced the concept of a cone E_b-metric space in which the cone contains semi-interior points. Building on this framework, the authors investigated several fixed point theorems, including Banach-type, Kannan-type, and Chatterjia-type results. It is important to note that the contraction constants for the Banach-type and Kannan-type fixed point theorems are given, respectively, by

$$0 < q < \frac{1}{s} \text{ and } 0 < q, q(1-q) < \frac{1}{s}, q \neq \frac{1}{s}.$$

More recently, Djedid et al. (2025) established a Hardy-Rogers-type fixed point theorem in cone E_b metric spaces. However, the proof of this theorem contains an error (see Theorem 3.1). It should be emphasized that Du's approach (Du, 2010) does not extend to cone E_b-metric spaces in which the cones have empty interiors but contain semi-interior points. Consequently, investigating fixed-point theorems in cone E_b-metric spaces with cones possessing empty interiors yet containing semi-interior points are a genuine extension.

First, we recall the following definitions that are used in this paper.

Definition 1.1. (Huang & Radenovic, 2016; Xu et al., 2022). Let E be a vector space over $\mathbb{R}$ and $P \subset E$.

(1) P satisfies the following conditions

(a) P is non-empty, closed, and $0 \in P$ where 0 is the null element of E.

(b) $\alpha P + \beta P \subset P$ for all non-negative real numbers α, β.

(c) $P \cap (-P) = \{0\}$.

Then P is said to be a cone.

(2) A partial ordering $\preceq$ on E with respect to P is defined by $x \preceq y$, for $x, y \in E$ if and only if $y - x \in P$. We write $x \prec y$ to indicate that $x \preceq y$ but $x \neq y$, and $x \ll y$ to indicate that $y - x \in \text{int}P$ where $\text{int}P$ is denoted by the interior of P.

(3) If $\text{int}P \neq \emptyset$, then P is said to be solid. On the other hand, if $\text{int}P = \emptyset$, then P is said to be non-solid.

(4) If there exists a number $\mu > 0$ such that for all $x, y \in \mathcal{A}, 0 \preceq x \preceq y, \|x\| \leq \mu\|y\|$, then P is said to be normal. The least positive number satisfying the above inequality is said to be a normal constant of P.

Throughout this paper, we assume that E is an ordered normed space with the zero element $0, P$ is a cone in $E, U = \{x \in E: \|x\| \leq 1\}$ is the closed unit ball of E, and $U_+ = U \cap E$ is the positive part of U.

Definition 1.2. Let $x, y \in P$. Then

(1) x is said to be a semi-interior point of P if there exists $k > 0$ such that $x - kU_+ \subset P$.

(2) $\hat{P}$ denotes the set of all the semi-interior points of P.

(3) We write $x \lll y$ to indicate that $y - x \in \hat{P}$.

It is clear that interior points of P are semi-interior points of P. However, the converse does not hold, see (Basile, 2017, Example 2.5).

Definition 1.3. (Boriceanu, 2009). Let X be a nonempty set, $s \geq 1, P \subset E$ with $\hat{P} \neq \emptyset$, and a function $d: X \times X \to P$ satisfy for each $x, y, z \in X$,

(1) $d(x, y) = 0$ if and only if $x = y$.

(2) $d(x, y) = d(y, x)$.

(3) $d(x, y) \preceq s(d(x, z) + d(z, y))$.

Then d is said to be a cone E_b-metric and (X, d, s) is said to be a cone E_b-metric space.

Definition 1.4. (Huang, 2019). Let P be a cone in E with $\hat{P} \neq \emptyset$ and $\{u_n\} \subset P$. Then $\{u_n\}$ is said to be an e-sequence if for each $0 \lll e$, there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0$, we have $u_n \lll e$.

It is clear that if $u_n \to 0$ as $n \to \infty$ then $\{u_n\}$ is an e-sequence.

Lemma 1.5. Let P be a cone in E with $\hat{P} \neq \emptyset$ and $u \in P$. If for each $0 \lll e, u \lll e$, then $u = 0$.

Definition 1.6. (Djedid et al., 2023). Let (X, d, s) be a cone E_b-metric space, $\{x_n\}$ be a sequence in X, and $x \in X$. Then

(1) $\{x_n\}$ is said to be e-convergent to x if for every $0 \lll e$, there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x) \lll e$ for all $n \geq n_0$.

(2) $\{x_n\}$ is said to be an e-Cauchy if for every $0 \lll e$, there exists $n_0 \in \mathbb{N}$ such that $d(x_n, x_m) \lll e$ for all $n, m \geq n_0$.

(3) (X, d, s) is said to be e-complete if every e-Cauchy sequence in (X, d, s) is e-convergent in (X, d, s).

Proposition 1.7. (Huang, 2019). Let $x, y, z \in P$. Then

(1) If $x \lll y$, then $x \preceq y$.

(2) If for each $e \in \hat{P}, 0 \preceq x \lll e$, then $x = 0$.

(3) If there exists $0 \leq \lambda < 1$ and $x \in P$ such that $x \preceq \lambda x$, then $x = 0$.

Proposition 1.8. (Huang, 2019). Let $\{x_n\}, \{y_n\} \subset P$ and $\alpha, \lambda \in \mathbb{R}_+$. Then

(1) If $x_n \to 0$ as $n \to \infty$, then $\{x_n\}$ is an e-sequence.

(2) If $\{x_n\}$ and $\{y_n\}$ are two e-sequences, then $\{\alpha x_n + \lambda y_n\}$ is also an e-sequence.

(3) If $x_n \preceq y_n$ for all $n \in \mathbb{N}$ and $\{y_n\}$ is an e-sequence, then $\{x_n\}$ is an e-sequence.

(4) If $\lambda < 1, \{y_n\}$ is an e-sequence, and $x_n \preceq \lambda x_{n-1} + y_{n-1}$ for all $n \in \mathbb{N}$, then $\{x_n\}$ is an e-sequence.

Lemma 1.9. (Djedid et al., 2023; Djedid et al., 2025). Let (X, d, s) be a cone E_b-metric space, $q \in \left[0, \frac{1}{s}\right)$, and $\{x_n\}$ be a sequence in X such that for all $n \in \mathbb{N}$,

$$d(x_n, x_{n+1}) \preceq q d(x_{n-1}, x_n).$$

Then $\{x_n\}$ is e-Cauchy in (X, d, s).

Theorem 1.10. (Djedid et al., 2023). Let (X, d, s) be a cone E_b-metric space and $f: X \to X$ be a map. If there exists $q \in \left[0, \frac{1}{s}\right)$ such that for all $x, y \in X$,

$$d(fx, fy) \preceq q d(x, y). \quad (1.1)$$

Then f has a unique fixed point $x^* \in X$ and for all $x \in X$, $\lim_{n\to\infty} f^n x = x^*$.

Theorem 1.11. *(*Djedid et al., 2023). Let (X, d, s) be a cone E_b-metric space and $f: X \to X$ be a map. If there exists $0 < q, \frac{q}{1-q} < \frac{1}{s}$, and $q \neq \frac{1}{s}$ such that for all $x, y \in X$,

$$d(fx, fy) \preceq q[d(x, fx) + d(y, fy)].$$

Then f has a unique fixed point $x^* \in X$ and for all $x \in X$, $\lim_{n\to\infty} f^n x = x^*$.

Theorem 1.12. (Djedid et al., 2025). Let (X, d, s) be a cone E_b-metric space and $f: X \to X$ be a map. If there exists $q_i > 0$ for $i = 1,2,3,4,5$ such that $q_1 + q_2 + q_3 + s(q_4 + q_5) < 1$ and $q_2 + q_5 < \frac{1}{s^2}$, and for all $x, y \in X$,

$$d(fx, fy) \preceq q_1 d(x, y) + q_2 d(x, fx) + q_3 d(y, fy) + q_4 d(x, fy) + q_5 d(y, fx).$$

Then f has a unique fixed point $x^* \in X$ and for all $x \in X$, $\lim_{n\to\infty} f^n x = x^*$.

Note that in Djedid et al. (2025), the authors established the condition for an e-Cauchy with $q \in \left[0, \frac{1}{s}\right)$, see Lemma 1.9. However, in the proof of (Djedid et al., 2025, Theorem 1), the authors applied Lemma 1.9 with $q < 1$ to prove the sequence $\{x_n\}$ is an e-Cauchy, see (Djedid et al., 2025, page 12). This is incorrect.

2. Main results

First, we establish the conditions for a sequence in cone E_b-metric spaces to be e-Cauchy.

Theorem 2.1. Let (X, d, s) be an e-complete cone E_b-metric space with a cone $P, \hat{P} \neq \emptyset$, $q \in [0,1)$, and $\{x_n\} \subset X$ such that for all $n \in \mathbb{N}$,

$$d(x_{n+1}, x_{n+2}) \preceq q d(x_n, x_{n+1}).$$

Then $\{x_n\}$ is an e-Cauchy sequence in X.

Proof. For all $n \in \mathbb{N}$, we have

$$d(x_{n+1}, x_{n+2}) \preceq qd(x_n, x_{n+1}) \preceq q^2 d(x_{n-1}, x_n) \preceq \cdots \preceq q^{n+1} d(x_0, x_1). \qquad (2.1)$$

For $m \in \mathbb{N}$, we denote $p = [\log_2 m]$ which is the integer part of $\log_2 m$. For all $l, m \in \mathbb{N}$, we get

$$d(x_{l+1}, x_{l+m}) \preceq \sum_{n=1}^{p} s^n d(x_{l+2^{n-1}}, x_{l+2^n}) + s^{p+1} d(x_{l+2^p}, x_{l+m}) \qquad (2.2)$$

where $\sum_{n=1}^{p} s^n d(x_{l+2^{n-1}}, x_{l+2^n})$ is assumed to be 0 if $p = 0$.

Note that $m \leq 2^{p+1}$. From (2.2), we obtain

$$\begin{aligned} d(x_{l+1}, x_{l+m}) &\preceq \sum_{n=1}^{p} s^{2n} \sum_{i=l}^{l+2^{n-1}-1} d(x_{2^{n-1}+i}, x_{2^{n-1}+i+1}) + s^{2(p+1)} \sum_{i=l}^{l+m-2^p-1} d(x_{2^p+i}, x_{2^p+i+1}) \\ &\preceq \sum_{n=1}^{p} s^{2n} \sum_{i=l}^{l+2^{n-1}-1} d(x_{2^{n-1}+i}, x_{2^{n-1}+i+1}) + s^{2(p+1)} \sum_{i=l}^{l+2^p-1} d(x_{2^p+i}, x_{2^p+i+1}) \\ &= \sum_{n=1}^{p+1} s^{2n} \sum_{i=l}^{l+2^{n-1}-1} d(x_{2^{n-1}+i}, x_{2^{n-1}+i+1}). \end{aligned} \qquad (2.3)$$

From (2.1) and (2.3), we get

$$\begin{aligned} d(x_{l+1}, x_{l+m}) &\preceq \sum_{n=1}^{p+1} s^{2n} \sum_{i=0}^{2^{n-1}-1} q^{l+2^{n-1}+i} d(x_0, x_1) \\ &\preceq q^l \frac{d(x_0, x_1)}{1-q} \sum_{n=1}^{p+1} s^{2n} q^{2^{n-1}} \\ &= q^l \frac{d(x_0, x_1)}{1-q} \sum_{n=1}^{p+1} q^{2^{n-1} + 2n\log_q s}. \end{aligned} \qquad (2.4)$$

It is clear that $\lim_{n \to \infty} \left(2^{n-1} + 2n\log_q s - n\right) = \infty$. Then there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $2^{n-1} + 2n\log_q s - n \geq 1$. Hence, $q^{2^{n-1}+2n\log_q s} \leq q^{n+1}$. It implies that the series $\sum_{n=1}^{\infty} q^{2^{n-1}+2n\log_q s}$ is convergent. By (2.4), we have

$$d(x_{l+1}, x_{l+m}) \preceq \frac{d(x_0, x_1)}{1-q} q^l S \qquad (2.5)$$

where $S = \sum_{n=1}^{\infty} q^{2^{n-1}+2^n\log_q s}$.

For each $0 \lll e$, there exists $\alpha > 0$ such that $e - \alpha U_+ \subset P$. By (2.5), there exists $n_0 \in \mathbb{N}$ such that for all $l \geq n_0$,

$$\frac{d(x_0, x_1)}{1-q} q^l S \in \frac{\alpha}{2} U_+.$$

Hence

$$e-\frac{d(x_0,x_1)}{1-q}q^l S-\frac{\alpha}{2}U_+ \subset e-\alpha U_+ \subset P.$$

It implies that

$$e-\frac{d(x_0,x_1)}{1-q}q^l S \in \hat{P},$$

that is,

$$\frac{d(x_0,x_1)}{1-q}q^l S \lll e.$$

By (2.5), for all $n,m \geq n_0$, we have

$$d(x_{l+1},x_{l+m}) \preceq \frac{d(x_0,x_1)}{1-q}q^l S \lll e.$$

It implies that $\{x_n\}$ is e-Cauchy.

Remark 2.2. Theorem 2.1 generalizes several existing results (Djedid et al., 2025, Lemma 2; Huang, 2019, Proposition 15; Miculescu & Mihail, 2017, Lemma 2.2; Djedid et al., 2023, Lemma 2.11).

In the proof of Djedid et al. (2025, Theorem 1), an error appears. They applied Lemma 1.9 under the assumption of $0 \leq q < 1$ to prove that $\{x_n\}$ is an e-Cauchy sequence, whereas the actual assumption of Lemma 1.9 requires $0 \leq q < \frac{1}{s}$. Consequently, the argument provided for Djedid et al. (2025, Theorem 1) cannot be regarded as valid. By combining the proof technique of Djedid et al.(2025, Theorem 1) with Lemma 2.1, we can rectify the flaw in the original argument. Moreover, we replace the condition

$$q_2+q_5<\frac{1}{s^2}$$

with the stronger condition

$$sq_2+s^2q_5<1.$$

Consequently, the following corollary is presented logically and rigorously.

Corollary 2.3. (The Hardy-Rogers type fixed point theorem) Assume that

(1) (X,d,s) is an e-complete cone E_b-metric space.

(2) $f: X \to X$ is a Hardy-Rogers type contraction, that is, there exists $q_i \in \mathbb{R}$, $i = 1,2,3,4,5$ such that $\sum_{i=1}^{3} q_i + s(q_4+q_5) < 1, sq_2+s^2q_5<1$, and for all $x,y \in X$,

$$d(fx,fy) \preceq q_1 d(x,y)+q_2 d(x,fx)+q_3 d(y,fy)+q_4 d(x,fy)+q_5 d(y,fx).$$

Then f has a unique fixed point $x^* \in X$ and for all $x \in X$, $\lim_{n\to\infty} f^n x = x^*$.

Proof. The argument follows as in the proof of (Djedid et al., 2025, Theorem 1), except that Theorem 2.1 is applied instead of (Djedid et al., 2025, Lemma 2).

By letting $q_2=q_3=q_4=q_5=0$ in Corollary 2.3, we obtain the following corollary.

Corollary 2.4. (The Banach type fixed point theorem) Assume that

(1) (X, d, s) is an e-complete cone E_b-metric space and $f: X \to X$ is a map.

(2) There exists $q \in [0,1)$ such that for all $x, y \in X$,

$d(fx, fy) \leq qd(x, y)$.

Then f has a unique fixed point $x^* \in X$ and for all $x \in X$, $\lim_{n\to\infty} f^n x = x^*$.

Remark 2.5. Corollary 2.4 generalizes several existing results (Djedid et al., 2023, Theorem 3.1; Huang & Xu, 2013, Theorem 2.1; Nguyen & Vo, 2016, Theorem 2.1).

By letting $q_1 = q_4 = q_5 = 0$ in Corollary 2.3, we obtain the following corollary.

Corollary 2.6. (The Kannan type fixed point theorem) Assume that

(1) (X, d, s) is an e -complete cone E_b-metric space and $f: X \to X$ is a map.

(2) There exists $q \in \left[0, \min\left\{\frac{1}{2}, \frac{1}{s}\right\}\right)$ such that for all $x, y \in X$,

$d(fx, fy) \leq q[d(x, fx) + d(y, fy)]$.

Then f has a unique fixed point $x^* \in X$ and for all $x \in X$, $\lim_{n\to\infty} f^n x = x^*$.

Remark 2.7. Corollary 2.6 generalizes results from Djedid et al. (2023, Theorem 3.3) and Lu et al. (2021, Theorem 2.2).

3. Conclusion

In this paper, by relaxing the assumption $k \in [0, \frac{1}{s})$ to $k \in [0,1)$ in (1.1), we have provided sufficient conditions ensuring that sequences in a cone E_b-metric space, with cones containing interior points, are e-Cauchy. Based on this result, some fixed-point theorems in cone E_b-metric spaces have been established. In addition, errors appearing in Djedid et al. (2025) have been corrected. The obtained results extend known theorems in the literature. Future research may further investigate these theorems in more general classes of generalized metric spaces.

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MỘT SỐ ĐỊNH LÍ ĐIỂM BẤT ĐỘNG TRONG KHÔNG GIAN E_b-METRIC NÓN

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TÓM TẮT

Lí thuyết điểm cố định đóng vai trò cơ bản trong toán học và có ứng dụng rộng rãi trong nhiều lĩnh vực khoa học. Trong những năm gần đây, nhiều nghiên cứu đã tập trung vào việc mở rộng các kết quả điểm cố định cổ điển sang các không gian metric tổng quát, trong đó không gian không gian E_b-metric nón nhận được sự chú ý đặc biệt. Trong bài báo này, chúng tôi sử dụng phương pháp của Miculescu và Mihail (2017) để thiết lập các điều kiện đủ để các dãy trong không gian E_b-metric nón, với các nón chứa các điểm bán trong, là dãy e-Cauchy. Kết quả của chúng tôi mở rộng công trình của Miculescu và Mihail (2017) trong không gian E_b-metric nón và tổng quát hóa hơn nữa các phát hiện của Zahia et al. (2023) và Djedid et al. (2025) bằng cách nới lỏng hệ số co từ $0 \leq q < \frac{1}{s}$ đến $0 \leq q < 1$. Như là một hệ quả, chúng tôi suy ra định lí điểm cố định kiểu Hardy-Rogers, định lí điểm cố định kiểu Banach và định lí điểm cố định kiểu Kannan trong không gian E_b-metric nón. Các kết quả thu được không chỉ khái quát hóa các định lí đã biết trước đây mà còn làm phong phú thêm lí thuyết điểm cố định trong không gian metric tổng quát. Hơn nữa, những đóng góp này mở ra một hướng nghiên cứu mới cho các ứng dụng trong phân tích phi tuyến, phương trình vi phân và tối ưu hóa, từ đó nhấn mạnh tầm quan trọng của không gian E_b-metric nón trong việc giải quyết các vấn đề toán học phức tạp.

Từ khóa: không gian E_b-metric nón; điểm bất động; điểm bán trong